PART I

CHAPTER 1
Functions and Their Graphs

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Section 1.1 Rectangular Coordinates

- You should be able to use the point-plotting method of graphing.
- You should be able to find x- and y-intercepts.
  (a) To find the x-intercepts, let $y = 0$ and solve for x.
  (b) To find the y-intercepts, let $x = 0$ and solve for y.
- You should be able to test for symmetry.
  (a) To test for x-axis symmetry, replace y with $-y$.
  (b) To test for y-axis symmetry, replace x with $-x$.
  (c) To test for origin symmetry, replace x with $-x$ and y with $-y$.
- You should know the standard equation of a circle with center (h, k) and radius r:
  $(x - h)^2 + (y - k)^2 = r^2$

Vocabulary Check

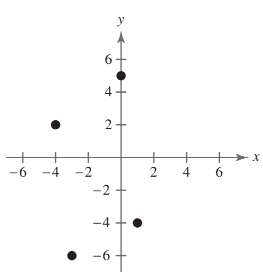
1. (a) **v** horizontal real number line
(b) **vi** vertical real number line
(c) **i** point of intersection of vertical axis and horizontal axis
(d) **iv** four regions of the coordinate plane
(e) **iii** directed distance from the y-axis
(f) **ii** directed distance from the x-axis

2. Cartesian **3.** Distance Formula **4.** Midpoint Formula

1. A: $(2, 6)$, B: $(-6, -2)$, C: $(4, -4)$, D: $(-3, 2)$

2. A: $\left(\frac{3}{2}, -4\right)$; B: $(0, -2)$; C: $\left(-3, \frac{5}{2}\right)$, D: $(-6, 0)$

3.



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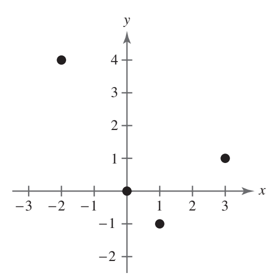

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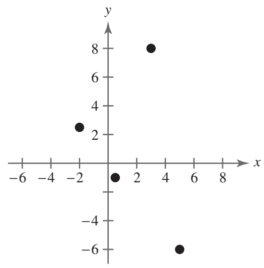

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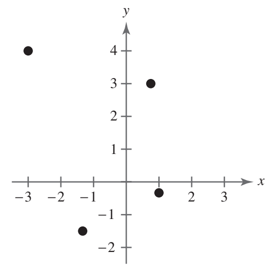


7. $(-3, 4)$

8. $(4, -8)$

9. $(-5, -5)$

10. $(-12, 0)$

11. $x > 0$ and $y < 0$ in Quadrant IV.

12. $x < 0$ and $y < 0$ in Quadrant III.

13. $x = -4$ and $y > 0$ in Quadrant II.

14. $x > 2$ and $y = 3$ in Quadrant I.

15. $y < -5$ in Quadrants III and IV.

16. $x > 4$ in Quadrants I and IV.

17. $(x, -y)$ is in the second Quadrant means that (x, y) is in Quadrant III.

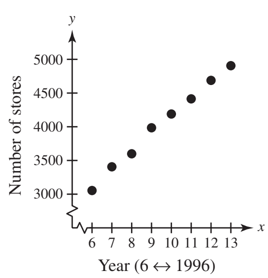
18. If $(-x, y)$ is in Quadrant IV, then (x, y) must be in Quadrant III.

19. (x, y), $xy > 0$ means x and y have the same signs. This occurs in Quadrants I and III.

20. If $xy < 0$, then x and y have opposite signs. This happens in Quadrants II and IV.

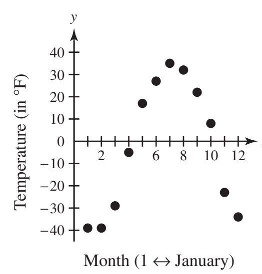
21.

Year, x	Number of stores, y
1996	3054
1997	3406
1998	3599
1999	3985
2000	4189
2001	4414
2002	4688
2003	4906



22.

Month, x	Temperature, y
1	-39
2	-39
3	-29
4	-5
5	17
6	27
7	35
8	32
9	22
10	8
11	-23
12	-34



23. $d = |5 - (-3)| = 8$

24. $d = |1 - 8| = |-7| = 7$

25. $d = |2 - (-3)| = 5$

26. $d = |-4 - 6|$

$= |-10| = 10$

27. (a) The distance between $(0, 2)$ and $(4, 2)$ is 4.

The distance between $(4, 2)$ and $(4, 5)$ is 3.

The distance between $(0, 2)$ and $(4, 5)$ is

$\sqrt{(4-0)^2 + (5-2)^2} = \sqrt{16+9} = \sqrt{25} = 5.$

(b) $4^2 + 3^2 = 16 + 9 = 25 = 5^2$

28. (a) $(1, 0), (13, 5)$

$$\text{Distance} = \sqrt{(13-1)^2 + (5-0)^2} = \sqrt{12^2 + 5^2} = \sqrt{169} = 13$$

$(13, 5), (13, 0)$

$$\text{Distance} = |5 - 0| = |5| = 5$$

$(1, 0), (13, 0)$

$$\text{Distance} = |1 - 13| = |-12| = 12$$

(b) $5^2 + 12^2 = 25 + 144 = 169 = 13^2$

29. (a) The distance between $(-1, 1)$ and $(9, 1)$ is 10.

The distance between $(9, 1)$ and $(9, 4)$ is 3.

The distance between $(-1, 1)$ and $(9, 4)$ is

$$\sqrt{(9-(-1))^2 + (4-1)^2} = \sqrt{100+9} = \sqrt{109}.$$

(b) $10^2 + 3^2 = 109 = \left(\sqrt{109}\right)^2$

30. (a) $(1, 5), (5, -2)$

$$\text{Distance} = \sqrt{(1-5)^2 + (5-(-2))^2} = \sqrt{(-4)^2 + (7)^2} = \sqrt{16+49} = \sqrt{65}$$

$(1, 5), (1, -2)$

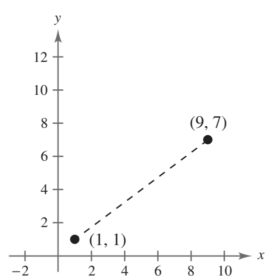
$$\text{Distance} = |5 - (-2)| = |5 + 2| = |7| = 7$$

$(1, -2), (5, -2)$

$$\text{Distance} = |1 - 5| = |-4| = 4$$

(b) $4^2 + 7^2 = 16 + 49 = 65 = \left(\sqrt{65}\right)^2$

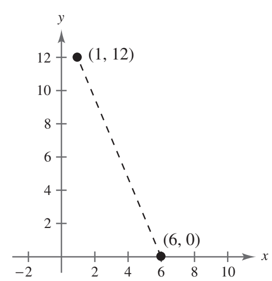
31. (a)



(b) $d = \sqrt{(9-1)^2 + (7-1)^2} = \sqrt{64+36} = 10$

(c) $\left(\dfrac{9+1}{2}, \dfrac{7+1}{2}\right) = (5, 4)$

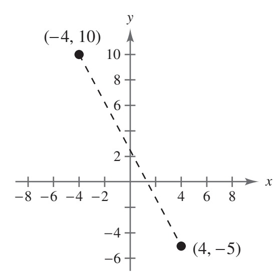
32. (a)



(b) $d = \sqrt{(1-6)^2 + (12-0)^2} = \sqrt{25+144} = 13$

(c) $\left(\dfrac{1+6}{2}, \dfrac{12+0}{2}\right) = \left(\dfrac{7}{2}, 6\right)$

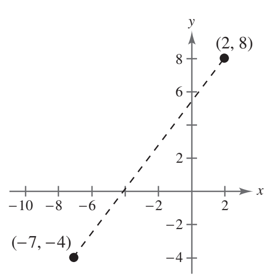
33. (a)



(b) $d = \sqrt{(4+4)^2 + (-5-10)^2} = \sqrt{64+225} = 17$

(c) $\left(\dfrac{4-4}{2}, \dfrac{-5+10}{2}\right) = \left(0, \dfrac{5}{2}\right)$

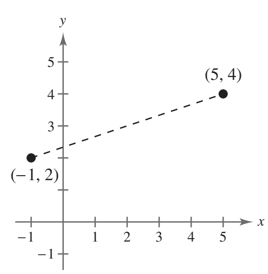
34. (a)



(b) $d = \sqrt{(-7-2)^2 + (-4-8)^2} = \sqrt{81+144} = 15$

(c) $\left(\dfrac{-7+2}{2}, \dfrac{-4+8}{2}\right) = \left(-\dfrac{5}{2}, 2\right)$

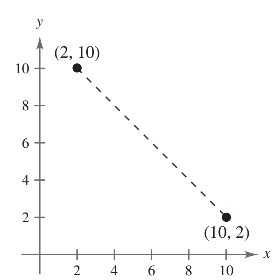
35. (a)



(b) $d = \sqrt{(5+1)^2 + (4-2)^2} = \sqrt{36+4} = 2\sqrt{10}$

(c) $\left(\dfrac{-1+5}{2}, \dfrac{2+4}{2}\right) = (2, 3)$

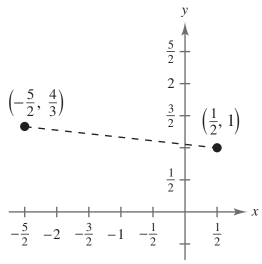
36. (a)



(b) $d = \sqrt{(2-10)^2 + (10-2)^2} = \sqrt{64+64} = 8\sqrt{2}$

(c) $\left(\dfrac{2+10}{2}, \dfrac{10+2}{2}\right) = (6, 6)$

37. (a)

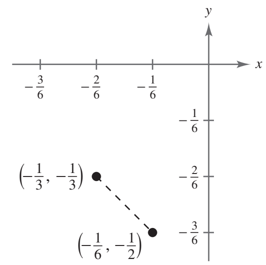


(b) $d = \sqrt{\left(\frac{1}{2} + \frac{5}{2}\right)^2 + \left(1 - \frac{4}{3}\right)^2}$

$= \sqrt{9 + \frac{1}{9}} = \frac{\sqrt{82}}{3}$

(c) $\left(\frac{-(5/2) + (1/2)}{2}, \frac{(4/3) + 1}{2}\right) = \left(-1, \frac{7}{6}\right)$

38. (a)

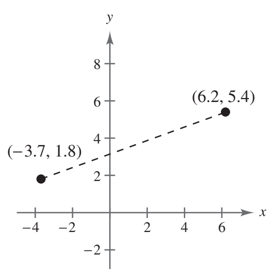


(b) $d = \sqrt{\left(-\frac{1}{3} + \frac{1}{6}\right)^2 + \left(-\frac{1}{3} + \frac{1}{2}\right)^2}$

$= \sqrt{\frac{1}{36} + \frac{1}{36}} = \frac{\sqrt{2}}{6}$

(c) $\left(\frac{-\frac{1}{3} + \left(-\frac{1}{6}\right)}{2}, \frac{-\frac{1}{3} + \left(-\frac{1}{2}\right)}{2}\right) = \left(-\frac{1}{4}, -\frac{5}{12}\right)$

39. (a)

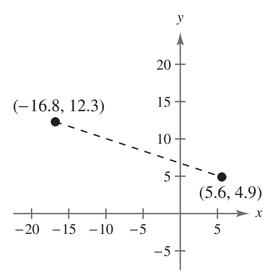


(b) $d = \sqrt{(6.2 + 3.7)^2 + (5.4 - 1.8)^2}$

$= \sqrt{98.01 + 12.96} = \sqrt{110.97}$

(c) $\left(\frac{6.2 - 3.7}{2}, \frac{5.4 + 1.8}{2}\right) = (1.25, 3.6)$

40. (a)



(b) $d = \sqrt{(-16.8 - 5.6)^2 + (12.3 - 4.9)^2}$

$= \sqrt{501.76 + 54.76} = \sqrt{556.52}$

(c) $\left(\frac{-16.8 + 5.6}{2}, \frac{12.3 + 4.9}{2}\right) = (-5.6, 8.6)$

41. $d_1 = \sqrt{(4 - 2)^2 + (0 - 1)^2} = \sqrt{5}$

$d_2 = \sqrt{(4 + 1)^2 + (0 + 5)^2} = \sqrt{50}$

$d_3 = \sqrt{(2 + 1)^2 + (1 + 5)^2} = \sqrt{45}$

$\left(\sqrt{5}\right)^2 + \left(\sqrt{45}\right)^2 = \left(\sqrt{50}\right)^2$

42. $d_1 = \sqrt{(1 - 3)^2 + (-3 - 2)^2} = \sqrt{4 + 25} = \sqrt{29}$

$d_2 = \sqrt{(3 + 2)^2 + (2 - 4)^2} = \sqrt{25 + 4} = \sqrt{29}$

$d_3 = \sqrt{(1 + 2)^2 + (-3 - 4)^2} = \sqrt{9 + 49} = \sqrt{58}$

$d_1 = d_2$

43. Since $x_m = \frac{x_1 + x_2}{2}$ and $y_m = \frac{y_1 + y_2}{2}$ we have:

$2x_m = x_1 + x_2$ $\quad$ $2y_m = y_1 + y_2$

$2x_m - x_1 = x_2$ $\quad$ $2y_m - y_1 = y_2$

Thus, $(x_2, y_2) = (2x_m - x_1, 2y_m - y_1)$.

44. (a) $(x_2, y_2) = (2x_m - x_1, 2y_m - y_1)$

$= (2 \cdot 4 - 1, 2(-1) - (-2)) = (7, 0)$

(b) $(x_2, y_2) = (2x_m - x_1, 2y_m - y_1)$

$= (2 \cdot 2 - (-5), 2 \cdot 4 - 11) = (9, -3)$

45. The midpoint of the given line segment is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

The midpoint between (x_1, y_1) and $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ is $\left(\frac{x_1 + \frac{x_1 + x_2}{2}}{2}, \frac{y_1 + \frac{y_1 + y_2}{2}}{2}\right) = \left(\frac{3x_1 + x_2}{4}, \frac{3y_1 + y_2}{4}\right)$.

The midpoint between $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ and (x_2, y_2) is $\left(\frac{\frac{x_1 + x_2}{2} + x_2}{2}, \frac{\frac{y_1 + y_2}{2} + y_2}{2}\right) = \left(\frac{x_1 + 3x_2}{4}, \frac{y_1 + 3y_2}{4}\right)$.

Thus, the three points are

$$\left(\frac{3x_1 + x_2}{4}, \frac{3y_1 + y_2}{4}\right), \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right), \text{ and } \left(\frac{x_1 + 3x_2}{4}, \frac{y_1 + 3y_2}{4}\right).$$

46. (a) $\left(\frac{3x_1 + x_2}{4}, \frac{3y_1 + y_2}{4}\right) = \left(\frac{3 \cdot 1 + 4}{4}, \frac{3(-2) - 1}{4}\right)$

$= \left(\frac{7}{4}, -\frac{7}{4}\right)$

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{1 + 4}{2}, \frac{-2 - 1}{2}\right) = \left(\frac{5}{2}, -\frac{3}{2}\right)$

$\left(\frac{x_1 + 3x_2}{4}, \frac{y_1 + 3y_2}{4}\right) = \left(\frac{1 + 3 \cdot 4}{4}, \frac{-2 + 3(-1)}{4}\right)$

$= \left(\frac{13}{4}, -\frac{5}{4}\right)$

(b) $\left(\frac{3x_1 + x_2}{4}, \frac{3y_1 + y_2}{4}\right) = \left(\frac{3(-2) + 0}{4}, \frac{3(-3) + 0}{4}\right)$

$= \left(-\frac{3}{2}, -\frac{9}{4}\right)$

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-2 + 0}{2}, \frac{-3 + 0}{2}\right) = \left(-1, -\frac{3}{2}\right)$

$\left(\frac{x_1 + 3x_2}{4}, \frac{y_1 + 3y_2}{4}\right) = \left(\frac{-2 + 0}{4}, \frac{-3 + 0}{4}\right) = \left(-\frac{1}{2}, -\frac{3}{4}\right)$

47. $d = \sqrt{(42 - 18)^2 + (50 - 12)^2}$

$= \sqrt{24^2 + 38^2}$

$= \sqrt{2020}$

$= 2\sqrt{505}$

≈ 45 yards

48. Distance $= \sqrt{120^2 + 150^2}$

$= \sqrt{36{,}900}$

$= 30\sqrt{41}$

≈ 192.09 kilometers

The plane flies about 192 kilometers.

49. $\left(\frac{2001 + 2003}{2}, \frac{3433 + 4174}{2}\right) = (2002, 3803.5)$

In 2002, the sales for Big Lots was approximately \$3803.5 million.

50. $\frac{\$1987 + \$2800}{2} = \frac{\$4787}{2}$

$\approx \$2393.50$ million

51. $(-2 + 2, -4 + 5) = (0, 1)$

$(2 + 2, -3 + 5) = (4, 2)$

$(-1 + 2, -1 + 5) = (1, 4)$

52. $(-3 + 6, 6 - 3) = (3, 3)$

$(-5 + 6, 3 - 3) = (1, 0)$

$(-3 + 6, 0 - 3) = (3, -3)$

$(-1 + 6, 3 - 3) = (5, 0)$

53. $(-7 + 4, -2 + 8) = (-3, 6)$

$(-2 + 4, 2 + 8) = (2, 10)$

$(-2 + 4, -4 + 8) = (2, 4)$

$(-7 + 4, -4 + 8) = (-3, 4)$

54. $(5 - 10, 8 - 6) = (-5, 2)$

$(3 - 10, 6 - 6) = (-7, 0)$

$(7 - 10, 6 - 6) = (-3, 0)$

$(5 - 10, 2 - 6) = (-5, -4)$

55. The highest price of butter is approximately \$3.31 per pound. This occurred in 2001.

56. Price of butter in 1995 $\approx$ \$1.75

Highest price of butter = \$3.31 in 2001

$$\text{Percent change} = \frac{3.31 - 1.75}{1.75} \approx 89.1\%$$

57. $\left[\dfrac{2400 - 700}{700}\right](100) \approx 242.9\%$ increase

58. (a) Cost during Super Bowl XXVII (1993) $\approx$ \$850,000

Cost during Super Bowl XXIII (1989) $\approx$ \$700,000

Increase = \$850,000 − \$700,000 $\approx$ \$150,000

$$\text{Percent increase} = \frac{\$150{,}000}{\$700{,}000} \approx 0.214, \text{ or } 21.4\%$$

(b) Cost during Super Bowl XXXVII (2003) $\approx$ \$2,100,000

Increase = \$2,100,000 − \$850,000 = \$1,250,000

$$\text{Percent increase} = \frac{\$1{,}250{,}000}{\$850{,}000} \approx 1.47, \text{ or } 147\%$$

59. (a) The number of artists elected each year seems to be nearly steady except for the first few years. Between 6 and 8 artists will be elected in 2008.

(b) Elections for inclusion in the Rock and Roll Hall of Fame began in 1986.

60. (a) The minimum wage had the greatest increase in the 1990s.

(b) Minimum wage in 1990: \$3.80

Minimum wage in 1995: \$4.25

$$\text{Percent increase: } \left(\frac{\$4.25 - \$3.80}{\$3.80}\right)(100) \approx 11.8\%$$

Minimum wage in 1995: \$4.25

Minimum wage in 2000: \$5.15

$$\text{Percent increase: } \left(\frac{\$5.15 - \$4.25}{\$4.25}\right)(100) \approx 21.2\%$$

(c) \$5.15 + 0.212(\$5.15) $\approx$ \$6.24

(d) The political nature of the minimum wage makes it difficult to predict, but this does seem like a reasonable value.

61. (1996, 18,546), (2004, 21,900)

By Exercise 45 we have the following:

$$\left(\frac{3(1996) + 2004}{4}, \frac{3(18{,}546) + 21{,}900}{4}\right) = (1998, 19{,}384.5)$$

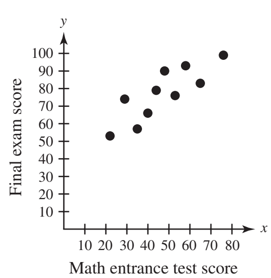
$$\left(\frac{1996 + 2004}{2}, \frac{18{,}546 + 21{,}900}{2}\right) = (2000, 20{,}223)$$

$$\left(\frac{1996 + 3(2004)}{4}, \frac{18{,}546 + 3(21{,}900)}{4}\right) = (2002, 21{,}061.5)$$

Year	Sales for Coca-Cola Company
1998	\$19,384.5 million
2000	\$20,223 million
2002	\$21,061.5 million

62. (a)

x	y
22	53
29	74
35	57
40	66
44	79
48	90
53	76
58	93
65	83
76	99



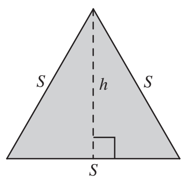
(b) The point $(65, 83)$ represents an entrance exam score of 65.

(c) No. There are many variables that will affect the final exam score.

63.
$$V = \frac{4}{3}\pi r^3$$
$$5.96 = \frac{4}{3}\pi r^3$$
$$17.88 = 4\pi r^3$$
$$\frac{17.88}{4\pi} = r^3$$
$$r = \sqrt[3]{\frac{4.47}{\pi}} \approx 1.12 \text{ inches}$$

64.
$$V = \pi r^2 h$$
$$h = \frac{V}{\pi r^2} = \frac{603.2}{\pi(2)^2} \approx 48 \text{ feet}$$

65.

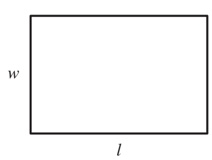


$$3S = 129$$
$$S = 43 \text{ centimeters}$$
$$h^2 + \left(\frac{S}{2}\right)^2 = S^2$$
$$h^2 = \frac{3S^2}{4}$$
$$h = \frac{\sqrt{3}S}{2}$$
$$A = \frac{1}{2}bh = \frac{1}{2}S\left(\frac{\sqrt{3}S}{2}\right) = \frac{\sqrt{3}S^2}{4}$$

When $S = 43$ centimeters,
$$A = \frac{\sqrt{3}(43)^2}{4} \approx 800.64 \text{ square centimeters.}$$

66.
$$S = \pi R\sqrt{R^2 + h^2}$$
$$1617 = (\pi)(14)\sqrt{14^2 + h^2}$$
$$\frac{1617}{14\pi} = \sqrt{196 + h^2}$$
$$\left(\frac{1617}{14\pi}\right)^2 = 196 + h^2$$
$$\left(\frac{1617}{14\pi}\right)^2 - 196 = h^2$$
$$\sqrt{\left(\frac{1617}{14\pi}\right)^2 - 196} = h$$
$$h \approx 33.995 \approx 34 \text{ centimeters}$$

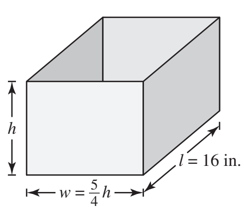
67. (a)



(b) $l = 1.5w$
$$P = 2l + 2w$$
$$= 2(1.5w) + 2w$$
$$= 5w$$

(c) $25 = 5w$
$$5 = w$$
Width: $w = 5$ meters
Length: $l = 1.5w = 7.5$ meters
Dimensions: 7.5 meters × 5 meters

68. (a)

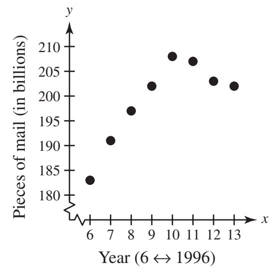


(b) $w = 1.25h = \frac{5}{4}h$
$$V = l \cdot w \cdot h = (16)\left(\tfrac{5}{4}h\right)(h)$$
$$V = 20h^2$$

(c) $V = 2000 = 20h^2$
$$100 = h^2 \implies h = 10 \text{ in.}$$
$$w = \left(\tfrac{5}{4}\right)(10) = \tfrac{25}{2} = 12.5 \text{ in.}$$
$$l = 16 \text{ in.}$$
Dimensions: 16 inches × 12.5 inches × 10 inches

69. (a)

Year, x	Pieces of mail, y (in billions)
1996	183
1997	191
1998	197
1999	202
2000	208
2001	207
2002	203
2003	202

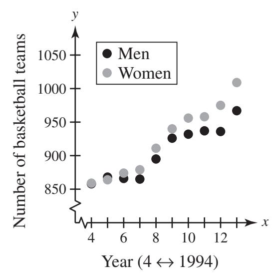


(b) The greatest decrease occurred in 2002.

(c) Answers will vary. Technology now enables us to transport information in ways other than by mail. The internet is one example.

70. (a)

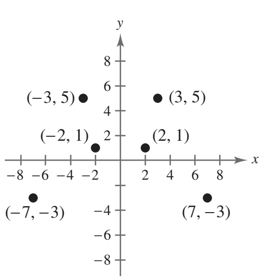
Year, x	Men's teams, M	Women's teams, W
1994	858	859
1995	868	864
1996	866	874
1997	865	879
1998	895	911
1999	926	940
2000	932	956
2001	937	958
2002	936	975
2003	967	1009



(b) In 1994, the number of men's and women's teams were nearly equal.

(c) In 2003, the difference between the number of teams was greatest: $1009 - 967 = 42$ teams.

71.



(a) The point is reflected through the y-axis.

(b) The point is reflected through the x-axis.

(c) The point is reflected through the origin.

72. (a) **First Set**

$d(A, B) = \sqrt{(2-2)^2 + (3-6)^2} = \sqrt{9} = 3$

$d(B, C) = \sqrt{(2-6)^2 + (6-3)^2} = \sqrt{16+9} = 5$

$d(A, C) = \sqrt{(2-6)^2 + (3-3)^2} = \sqrt{16} = 4$

Since $3^2 + 4^2 = 5^2$, A, B, and C are the vertices of a right triangle.

Second Set

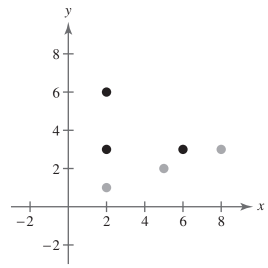
$d(A, B) = \sqrt{(8-5)^2 + (3-2)^2} = \sqrt{10}$

$d(B, C) = \sqrt{(5-2)^2 + (2-1)^2} = \sqrt{10}$

$d(A, C) = \sqrt{(8-2)^2 + (3-1)^2} = \sqrt{40}$

A, B, and C are the vertices of an isosceles triangle or are collinear: $\sqrt{10} + \sqrt{10} = 2\sqrt{10} = \sqrt{40}$.

(b)



First set: Not collinear

Second set: The points are collinear.

(c) If A, B, and C are collinear, then two of the distances will add up to the third distance.

73. False, you would have to use the Midpoint Formula 15 times.

74. True. Two sides of the triangle have lengths $\sqrt{149}$ and the third side has a length of $\sqrt{18}$.

75. No. It depends on the magnitude of the quantities measured.

76. Use the Midpoint Formula to prove the diagonals of the parallelogram bisect each other.

$$\left(\frac{b+a}{2}, \frac{c+0}{2}\right) = \left(\frac{a+b}{2}, \frac{c}{2}\right)$$

$$\left(\frac{a+b+0}{2}, \frac{c+0}{2}\right) = \left(\frac{a+b}{2}, \frac{c}{2}\right)$$

77. Since (x_0, y_0) lies in Quadrant II, $(x_0, -y_0)$ must lie in Quadrant III. Matches (b).

78. Since (x_0, y_0) lies in Quadrant II, $(-2x_0, y_0)$ must lie in Quadrant I. Matches (c).

79. Since (x_0, y_0) lies in Quadrant II, $\left(x_0, \frac{1}{2}y_0\right)$ must lie in Quadrant II. Matches (d).

80. Since (x_0, y_0) lies in Quadrant II, $(-x_0, -y_0)$ must lie in Quadrant IV. Matches (a).

81.
$$\begin{aligned} 2x + 1 &= 7x - 4 \\ -5x &= -5 \\ x &= 1 \end{aligned}$$

82.
$$\begin{aligned} \tfrac{1}{3}x + 2 &= 5 - \tfrac{1}{6}x \\ \tfrac{1}{3}x + \tfrac{1}{6}x &= 5 - 2 \\ \tfrac{1}{2}x &= 3 \\ x &= 6 \end{aligned}$$

83.
$$\begin{aligned} x^2 - 4x - 7 &= 0 \\ x^2 - 4x &= 7 \\ x^2 - 4x + 4 &= 7 + 4 \\ (x-2)^2 &= 11 \\ x - 2 &= \pm\sqrt{11} \\ x &= 2 \pm \sqrt{11} \end{aligned}$$

84.
$$\begin{aligned} 2x^2 + 3x - 8 &= 0 \\ x &= \frac{-3 \pm \sqrt{(3)^2 - (4)(2)(-8)}}{(2)(2)} \\ x &= \frac{-3 \pm \sqrt{9+64}}{4} \\ x &= \frac{-3 \pm \sqrt{73}}{4} \end{aligned}$$

85.
$$\begin{aligned} 3x + 1 &< 2(2 - x) \\ 3x + 1 &< 4 - 2x \\ 5x &< 3 \\ x &< \tfrac{3}{5} \end{aligned}$$

86.
$$\begin{aligned} 3x - 8 &\ge \tfrac{1}{2}(10x + 7) \\ 2(3x - 8) &\ge 10x + 7 \\ 6x - 16 &\ge 10x + 7 \\ -4x &\ge 23 \\ x &\le -\tfrac{23}{4} \end{aligned}$$

87. $|x - 18| < 4$

$-4 < x - 18 < 4$

$14 < x < 22$

88. $|2x + 15| \geq 11$

$2x + 15 \geq 11$ or $2x + 15 \leq -11$

$2x \geq 11 - 15$ $\quad$ $2x \leq -11 - 15$

$2x \geq -4$ $\quad$ $2x \leq -26$

$x \geq -2$ $\quad$ $x \leq -13$

Section 1.2 Graphs of Equations

You should know the following important facts about lines.

- The graph of $y = mx + b$ is a straight line. It is called a linear equation in two variables.
  - (a) The slope (steepness) is m.
  - (b) The y-intercept is $(0, b)$.
- The slope of the line through (x_1, y_1) and (x_2, y_2) is

  $$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise}}{\text{run}}.$$
- (a) If $m > 0$, the line rises from left to right.
  - (b) If $m = 0$, the line is horizontal.
  - (c) If $m < 0$, the line falls from left to right.
  - (d) If m is undefined, the line is vertical.
- Equations of Lines
  - (a) Slope-Intercept Form: $y = mx + b$
  - (b) Point-Slope Form: $y - y_1 = m(x - x_1)$
  - (c) Two-Point Form: $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
  - (d) General Form: $Ax + By + C = 0$
  - (e) Vertical Line: $x = a$
  - (f) Horizontal Line: $y = b$
- Given two distinct nonvertical lines

  $L_1: y = m_1x + b_1$ and $L_2: y = m_2x + b_2$
  - (a) L_1 is parallel to L_2 if and only if $m_1 = m_2$ and $b_1 \neq b_2$.
  - (b) L_1 is perpendicular to L_2 if and only if $m_1 = -1/m_2$.

Vocabulary Check

1. solution or solution point	**2.** graph	**3.** intercepts
4. y-axis	**5.** circle; (h, k); r	**6.** numerical

1. $y = \sqrt{x+4}$

(a) $(0, 2)$: $2 \stackrel{?}{=} \sqrt{0+4}$

$2 = 2$

Yes, the point *is* on the graph.

(b) $(5, 3)$: $3 \stackrel{?}{=} \sqrt{5+4}$

$3 = \sqrt{9}$

Yes, the point *is* on the graph.

2. $y = x^2 - 3x + 2$

(a) $(2, 0)$: $(2)^2 - 3(2) + 2 \stackrel{?}{=} 0$

$4 - 6 + 2 \stackrel{?}{=} 0$

$0 = 0$

Yes, the point *is* on the graph.

(b) $(-2, 8)$: $(-2)^2 - 3(-2) + 2 \stackrel{?}{=} 8$

$4 + 6 + 2 \stackrel{?}{=} 8$

$12 \neq 8$

No, the point *is not* on the graph.

3. $y = 4 - |x - 2|$

(a) $(1, 5)$: $5 \stackrel{?}{=} 4 - |1 - 2|$

$5 \neq 4 - 1$

No, the point *is not* on the graph.

(b) $(6, 0)$: $0 \stackrel{?}{=} 4 - |6 - 2|$

$0 = 4 - 4$

Yes, the point *is* on the graph.

4. $y = \frac{1}{3}x^3 - 2x^2$

(a) $\left(2, -\frac{16}{3}\right)$: $\frac{1}{3}(2)^3 - 2(2)^2 \stackrel{?}{=} -\frac{16}{3}$

$\frac{1}{3} \cdot 8 - 2 \cdot 4 \stackrel{?}{=} -\frac{16}{3}$

$\frac{8}{3} - 8 \stackrel{?}{=} -\frac{16}{3}$

$\frac{8}{3} - \frac{24}{3} \stackrel{?}{=} -\frac{16}{3}$

$-\frac{16}{3} = -\frac{16}{3}$

Yes, the point *is* on the graph.

(b) $(-3, 9)$: $\frac{1}{3}(-3)^3 - 2(-3)^2 \stackrel{?}{=} 9$

$\frac{1}{3}(-27) - 2(9) \stackrel{?}{=} 9$

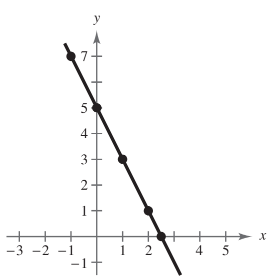
$-9 - 18 \stackrel{?}{=} 9$

$-27 \neq 9$

No, the point *is not* on the graph.

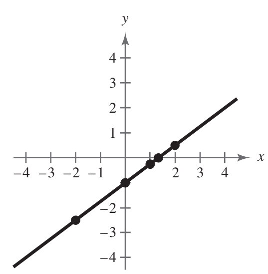
5. $y = -2x + 5$

x	-1	0	1	2	$\frac{5}{2}$
y	7	5	3	1	0
(x, y)	$(-1, 7)$	$(0, 5)$	$(1, 3)$	$(2, 1)$	$\left(\frac{5}{2}, 0\right)$



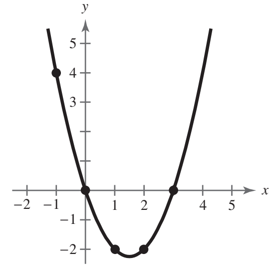
6. $y = \frac{3}{4}x - 1$

x	-2	0	1	$\frac{4}{3}$	2
y	$-\frac{5}{2}$	-1	$-\frac{1}{4}$	0	$\frac{1}{2}$
x, y	$\left(-2, -\frac{5}{2}\right)$	$(0, -1)$	$\left(1, -\frac{1}{4}\right)$	$\left(\frac{4}{3}, 0\right)$	$\left(2, \frac{1}{2}\right)$



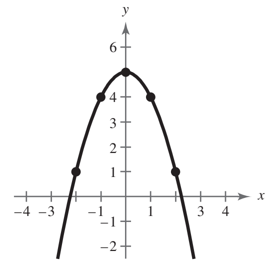
7. $y = x^2 - 3x$

x	-1	0	1	2	3
y	4	0	-2	-2	0
(x, y)	$(-1, 4)$	$(0, 0)$	$(1, -2)$	$(2, -2)$	$(3, 0)$



8. $5 - x^2$

x	-2	-1	0	1	2
y	1	4	5	4	1
x, y	$(-2, 1)$	$(-1, 4)$	$(0, 5)$	$(1, 4)$	$(2, 1)$



9. $y = 16 - 4x^2$

x-intercepts: $0 = 16 - 4x^2$

$4x^2 = 16$

$x^2 = 4$

$x = \pm 2$

$(-2, 0), (2, 0)$

y-intercept: $y = 16 - 4(0)^2 = 16$

$(0, 16)$

10. $y = (x + 3)^2$

x-intercept: $0 = (x + 3)^2$

$0 = x + 3$

$x = -3$

$(-3, 0)$

y-intercept: $y = (0 + 3)^2$

$y = 3^2$

$y = 9$

$(0, 9)$

11. $y = 5x - 6$

x-intercept: $0 = 5x - 6$

$6 = 5x$

$\frac{6}{5} = x$

$\left(\frac{6}{5}, 0\right)$

y-intercept: $y = 5(0) - 6 = -6$

$(0, -6)$

12. $y = 8 - 3x$

x-intercept: $0 = 8 - 3x$

$3x = 8$

$x = \frac{8}{3}$

$\left(\frac{8}{3}, 0\right)$

y-intercept: $y = 8 - 3(0) = 8$

$(0, 8)$

13. $y = \sqrt{x + 4}$

x-intercept: $0 = \sqrt{x + 4}$

$0 = x + 4$

$-4 = x$

$(-4, 0)$

y-intercept: $y = \sqrt{0 + 4} = 2$

$(0, 2)$

14. $y = \sqrt{2x - 1}$

x-intercept: $0 = \sqrt{2x - 1}$

$2x - 1 = 0$

$x = \frac{1}{2}$

$\left(\frac{1}{2}, 0\right)$

y-intercept: $y = \sqrt{2(0) - 1}$

$= \sqrt{-1}$ There is no real solution.

There is no y-intercept.

15. $y = |3x - 7|$

x-intercept: $0 = |3x - 7|$

$0 = 3x - 7$

$\frac{7}{3} = 0$

$\left(\frac{7}{3}, 0\right)$

y-intercept: $y = |3(0) - 7| = 7$

$(0, 7)$

16. $y = -|x + 10|$

x-intercept: $0 = -|x + 10|$

$x + 10 = 0$

$x = -10$

$(-10, 0)$

y-intercept: $y = -|0 + 10|$

$= -|10| = -10$

$(0, -10)$

17. $y = 2x^3 - 4x^2$

x-intercepts: $0 = 2x^3 - 4x^2$

$0 = 2x^2(x - 2)$

$x = 0$ or $x = 2$

$(0, 0), (2, 0)$

y-intercept: $y = 2(0)^3 - 4(0)^2$

$y = 0$

$(0, 0)$

18. $y = x^4 - 25$

x-intercept: $0 = x^4 - 25$

$x^4 = 25$

$x = \pm\sqrt[4]{5^2} = \pm\sqrt{5}$

$\left(\pm\sqrt{5}, 0\right)$

y-intercept: $y = (0)^4 - 25 = -25$

$(0, -25)$

19. $y^2 = 6 - x$

x-intercept: $0 = 6 - x$

$x = 6$

$(6, 0)$

y-intercepts: $y^2 = 6 - 0$

$y = \pm\sqrt{6}$

$\left(0, \sqrt{6}\right), \left(0, -\sqrt{6}\right)$

20. $y^2 = x + 1$

x-intercept: $0 = x + 1$

$x = -1$

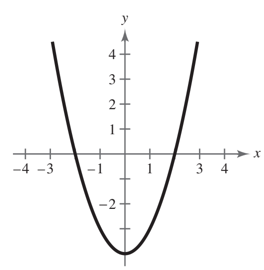
$(-1, 0)$

y-intercepts: $y^2 = 0 + 1$

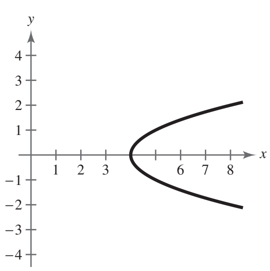
$y = \pm 1$

$(0, 1), (0, -1)$

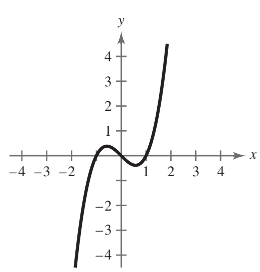
21. y-axis symmetry



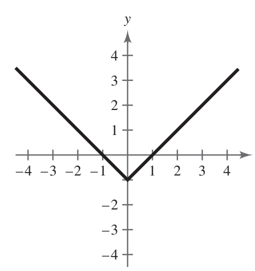
22.



23. Origin symmetry



24.



25. $x^2 - y = 0$

$(-x)^2 - y = 0 \Rightarrow x^2 - y = 0 \Rightarrow$ y-axis symmetry

$x^2 - (-y) = 0 \Rightarrow x^2 + y = 0 \Rightarrow$ No x-axis symmetry

$(-x)^2 - (-y) = 0 \Rightarrow x^2 + y = 0 \Rightarrow$ No origin symmetry

26. $x - y^2 = 0$

$x - (-y)^2 = 0$

$x - y^2 = 0$

x-axis symmetry

27. $y = x^3$

$y = (-x)^3 \implies y = -x^3 \implies$ No y-axis symmetry

$-y = x^3 \implies y = -x^3 \implies$ No x-axis symmetry

$-y = (-x)^3 \implies -y = -x^3 \implies y = x^3 \implies$ Origin symmetry

28. $y = x^4 - x^2 + 3$

$y = (-x)^4 - (-x)^2 + 3 \implies y = x^4 - x^2 + 3 \implies$ y-axis symmetry

$-y = x^4 - x^2 + 3 \implies y = -x^4 + x^2 - 3 \implies$ No x-axis symmetry

$-y = (-x)^4 - (-x)^2 + 3 \implies y = -x^4 + x^2 - 3 \implies$ No origin symmetry

29. $y = \dfrac{x}{x^2 + 1}$

$y = \dfrac{-x}{(-x)^2 + 1} \implies y = \dfrac{-x}{x^2 + 1} \implies$ No y-axis symmetry

$-y = \dfrac{x}{x^2 + 1} \implies y = \dfrac{-x}{x^2 + 1} \implies$ No x-axis symmetry

$-y = \dfrac{-x}{(-x)^2 + 1} \implies -y = \dfrac{-x}{x^2 + 1} \implies y = \dfrac{x}{x^2 + 1} \implies$ Origin symmetry

30. $y = \dfrac{1}{1 + x^2}$

$y = \dfrac{1}{1 + (-x)^2} \implies y = \dfrac{1}{1 + x^2} \implies$ y-axis symmetry

$-y = \dfrac{1}{1 + x^2} \implies y = \dfrac{-1}{1 + x^2} \implies$ No x-axis symmetry

$-y = \dfrac{1}{1 + (-x)^2} \implies y = \dfrac{-1}{1 + x^2} \implies$ No origin symmetry

31. $xy^2 + 10 = 0$

$(-x)y^2 + 10 = 0 \implies -xy^2 + 10 = 0 \implies$ No y-axis symmetry

$x(-y)^2 + 10 = 0 \implies xy^2 + 10 = 0 \implies$ x-axis symmetry

$(-x)(-y)^2 + 10 = 0 \implies -xy^2 + 10 = 0 \implies$ No origin symmetry

32. $xy = 4$

$(-x)y = 4 \implies xy = -4 \implies$ No y-axis symmetry

$x(-y) = 4 \implies xy = -4 \implies$ No x-axis symmetry

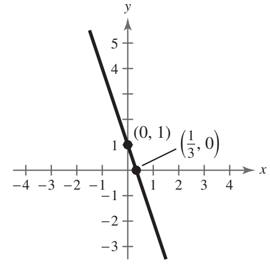
$(-x)(-y) = 4 \implies xy = 4 \implies$ Origin symmetry

33. $y = -3x + 1$

x-intercept: $\left(\frac{1}{3}, 0\right)$

y-intercept: $(0, 1)$

No axis or origin symmetry

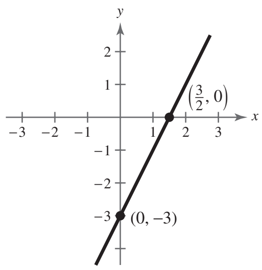


34. $y = 2x - 3$

x-intercept: $\left(\frac{3}{2}, 0\right)$

y-intercept: $(0, -3)$

No symmetry

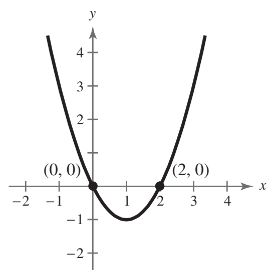


35. $y = x^2 - 2x$

Intercepts: $(0, 0)$, $(2, 0)$

No axis or origin symmetry

x	-1	0	1	2	3
y	3	0	-1	0	3

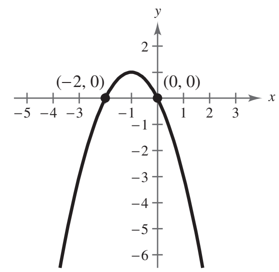


36. $y = -x^2 - 2x$

x-intercept: $(-2, 0)$, $(0, 0)$

y-intercept: $(0, 0)$

No symmetry

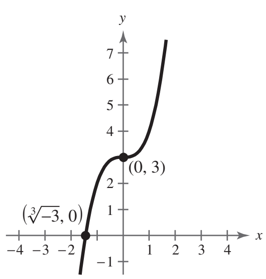


37. $y = x^3 + 3$

Intercepts: $(0, 3)$, $\left(\sqrt[3]{-3}, 0\right)$

No axis or origin symmetry

x	-2	-1	0	1	2
y	-5	2	3	4	11

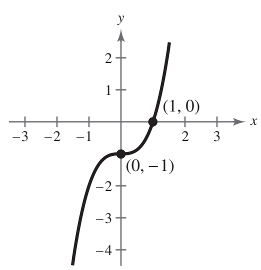


38. $y = x^3 - 1$

x-intercept: $(1, 0)$

y-intercept: $(0, -1)$

No symmetry



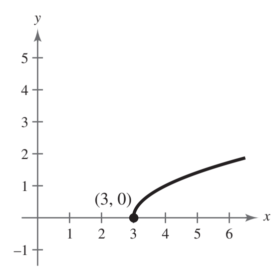
39. $y = \sqrt{x - 3}$

Domain: $[3, \infty)$

Intercept: $(3, 0)$

No axis or origin symmetry

x	3	4	7	12
y	0	1	2	3



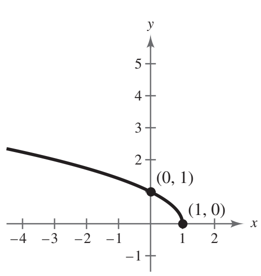
40. $y = \sqrt{1 - x}$

Domain: $(-\infty, 1]$

x-intercept: $(1, 0)$

y-intercept: $(0, 1)$

No symmetry

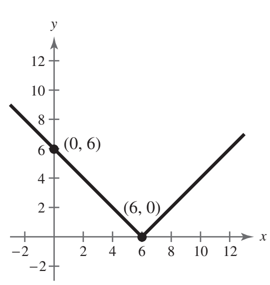


41. $y = |x - 6|$

Intercepts: $(0, 6)$, $(6, 0)$

No axis or origin symmetry

x	-2	0	2	4	6	8	10
y	8	6	4	2	0	2	4

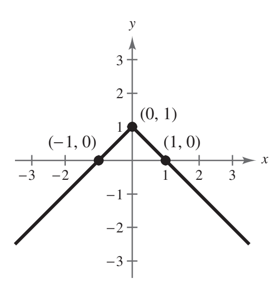


42. $y = 1 - |x|$

x-intercepts: $(\pm 1, 0)$

y-intercept: $(0, 1)$

y-axis symmetry

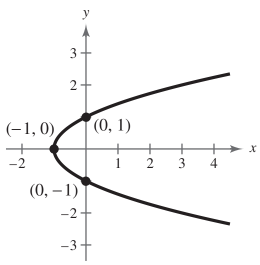


43. $x = y^2 - 1$

Intercepts: $(0, -1), (0, 1), (-1, 0)$

x-axis symmetry

x	-1	0	3
y	0	± 1	± 2

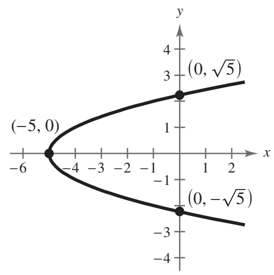


44. $x = y^2 - 5$

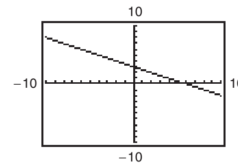
x-intercept: $(-5, 0)$

y-intercept: $\left(0, \pm\sqrt{5}\right)$

x-axis symmetry

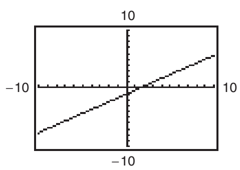


45. $y = 3 - \frac{1}{2}x$



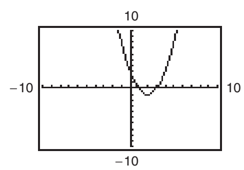
Intercepts: $(6, 0), (0, 3)$

46. $y = \frac{2}{3}x - 1$



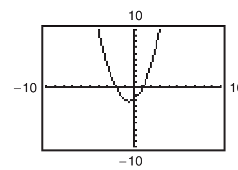
Intercepts: $(0, -1), \left(\frac{3}{2}, 0\right)$

47. $y = x^2 - 4x + 3$



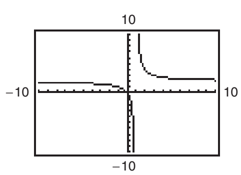
Intercepts: $(3, 0), (1, 0), (0, 3)$

48. $y = x^2 + x - 2$



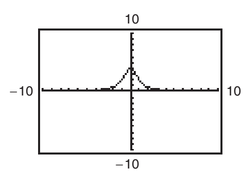
Intercepts: $(-2, 0), (1, 0), (0, -2)$

49. $y = \dfrac{2x}{x - 1}$



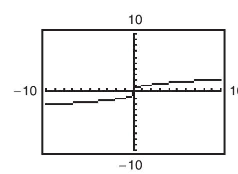
Intercept: $(0, 0)$

50. $y = \dfrac{4}{x^2 + 1}$



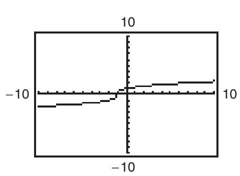
Intercept: $(0, 4)$

51. $y = \sqrt[3]{x}$



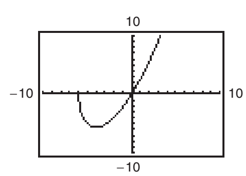
Intercept: $(0, 0)$

52. $y = \sqrt[3]{x + 1}$



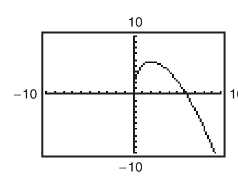
Intercepts: $(-1, 0), (0, 1)$

53. $y = x\sqrt{x + 6}$



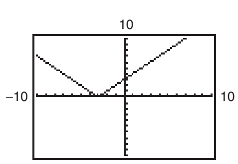
Intercepts: $(0, 0), (-6, 0)$

54. $y = (6 - x)\sqrt{x}$



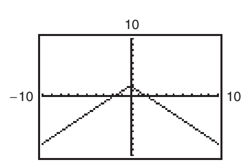
Intercepts: $(0, 0), (6, 0)$

55. $y = |x + 3|$



Intercepts: $(-3, 0), (0, 3)$

56. $y = 2 - |x|$



Intercepts: $(\pm 2, 0), (0, 2)$

57. Center: $(0, 0)$; radius: 4

Standard form:

$$(x - 0)^2 + (y - 0)^2 = 4^2$$

$$x^2 + y^2 = 16$$

58. $(x - 0)^2 + (y - 0)^2 = 5^2$

$x^2 + y^2 = 25$

59. Center: $(2, -1)$; radius: 4

Standard form:

$(x - 2)^2 + (y - (-1))^2 = 4^2$

$(x - 2)^2 + (y + 1)^2 = 16$

60. $(x - (-7))^2 + (y - (-4))^2 = 7^2$

$(x + 7)^2 + (y + 4)^2 = 49$

61. Center: $(-1, 2)$; solution point: $(0, 0)$

$(x - (-1))^2 + (y - 2)^2 = r^2$

$(0 + 1)^2 + (0 - 2)^2 = r^2 \implies 5 = r^2$

Standard form: $(x + 1)^2 + (y - 2)^2 = 5$

62. $r = \sqrt{(3 - (-1))^2 + (-2 - 1)^2}$

$= \sqrt{4^2 + (-3)^2} = \sqrt{25} = 5$

$(x - 3)^2 + (y - (-2))^2 = 5^2$

$(x - 3)^2 + (y + 2)^2 = 25$

63. Endpoints of a diameter: $(0, 0), (6, 8)$

Center: $\left(\dfrac{0 + 6}{2}, \dfrac{0 + 8}{2}\right) = (3, 4)$

$(x - 3)^2 + (y - 4)^2 = r^2$

$(0 - 3)^2 + (0 - 4)^2 = r^2 \implies 25 = r^2$

Standard form: $(x - 3)^2 + (y - 4)^2 = 25$

64. $r = \dfrac{1}{2}\sqrt{(-4 - 4)^2 + (-1 - 1)^2}$

$= \dfrac{1}{2}\sqrt{(-8)^2 + (-2)^2}$

$= \dfrac{1}{2}\sqrt{64 + 4}$

$= \dfrac{1}{2}\sqrt{68} = \left(\dfrac{1}{2}\right)(2)\sqrt{17} = \sqrt{17}$

Midpoint of diameter (center of circle):

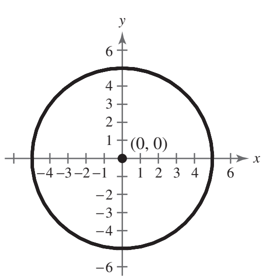
$\left(\dfrac{-4 + 4}{2}, \dfrac{-1 + 1}{2}\right) = (0, 0)$

$(x - 0)^2 + (y - 0)^2 = \left(\sqrt{17}\right)^2$

$x^2 + y^2 = 17$

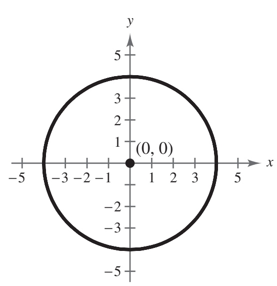
65. $x^2 + y^2 = 25$

Center: $(0, 0)$, radius: 5



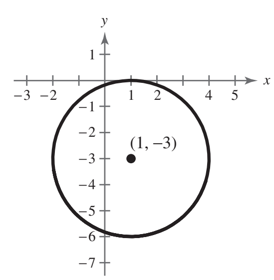
66. $x^2 + y^2 = 16$

Center: $(0, 0)$, radius: 4



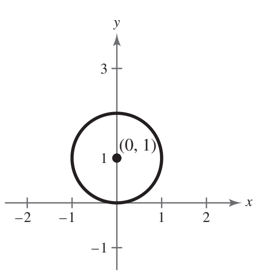
67. $(x - 1)^2 + (y + 3)^2 = 9$

Center: $(1, -3)$, radius: 3



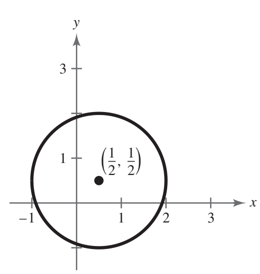
68. $x^2 + (y - 1)^2 = 1$

Center: $(0, 1)$, radius: 1



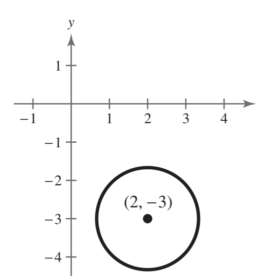
69. $\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{9}{4}$

Center: $\left(\frac{1}{2}, \frac{1}{2}\right)$, radius: $\frac{3}{2}$

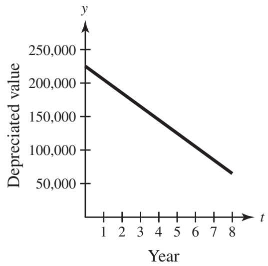


70. $(x - 2)^2 + (y + 3)^2 = \frac{16}{9}$

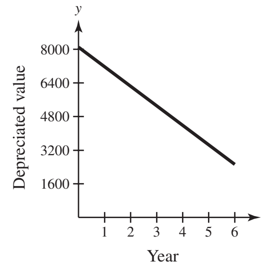
Center: $(2, -3)$, radius: $\frac{4}{3}$



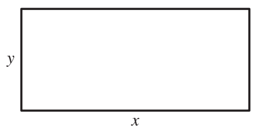
71. $y = 225{,}000 - 20{,}000t,\ 0 \le t \le 8$



72. $y = 8100 - 929t,\ 0 \le t \le 6$



73. (a)



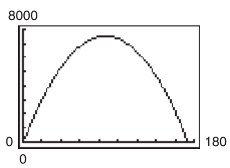
(b) $2x + 2y = \frac{1040}{3}$

$$2y = \frac{1040}{3} - 2x$$

$$y = \frac{520}{3} - x$$

$$A = xy = x\left(\frac{520}{3} - x\right)$$

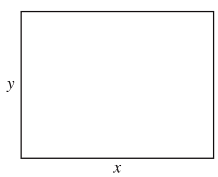
(c)



(d) When $x = y = 86\frac{2}{3}$ yards, the area is a maximum of $7511\frac{1}{9}$ square yards.

(e) A regulation NFL playing field is 120 yards long and $53\frac{1}{3}$ yards wide. The actual area is 6400 square yards.

74. (a)



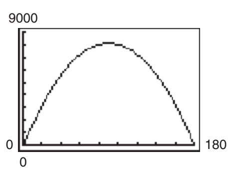
(b) $P = 360$ meters so:

$$2x + 2y = 360$$

$$w = y = 180 - x$$

$$A = lw = x(180 - x)$$

(c)



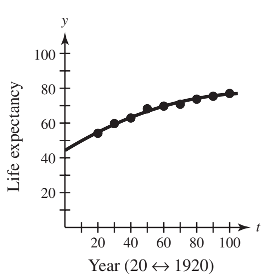
(d) $x = 90$ and $y = 90$

A square will give the maximum area of 8100 square meters.

(e) The dimensions of a Major League Soccer field can vary between 110 and 120 yards in length and between 70 and 80 yards in width.

75. $y = -0.0025t^2 + 0.574t + 44.25,\ 20 \le t \le 100$

(a) and (b)



(c) For the year 1948, let $t = 48$: $y \approx 66.0$ years.

(d) For the year 2005, let $t = 105$: $y \approx 77.0$ years.

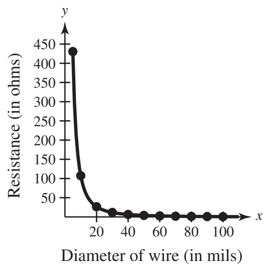
For the year 2010, let $t = 110$: $y \approx 77.1$ years.

(e) No. The graph reaches a maximum of $y \approx 77.2$ years when $t \approx 114.8$, or during the year 2014. After this time, the model has life expectancy decreasing, which is not realistic.

76. (a)

x	5	10	20	30	40	50	60	70	80	90	100
y	430.43	107.33	26.56	11.60	6.36	3.94	2.62	1.83	1.31	0.96	0.71

(b)



(c) When $x = 85.5$,

$$y = \frac{10{,}770}{85.5^2} - 0.37 = 1.10327.$$

(d) As the diameter of the wire increases, the resistance decreases.

77. False. A graph is symmetric with respect to the x-axis if, whenever (x, y) is on the graph, $(x, -y)$ is also on the graph.

78. True. The graph can have no intercepts, one, two or many. For example, a circle centered at the origin has two y-intercepts. A circle of radius 1, centered at $(7, 7)$, has no y-intercepts.

79. The viewing window is incorrect. Change the viewing window. Examples will vary. For example, $y = x^2 + 20$ will not appear in the standard window setting.

80. $y = ax^2 + bx^3$

(a) $y = a(-x)^2 + b(-x)^3$

$= ax^2 - bx^3$

To be symmetric with respect to the y-axis; a can be any non-zero real number, b must be zero.

(b) $-y = a(-x)^2 + b(-x)^3$

$-y = ax^2 - bx^3$

$y = -ax^2 + bx^3$

To be symmetric with respect to the origin; a must be zero, b can be any non-zero real number.

81. $9x^5 + 4x^3 - 7$

Terms: $9x^5, 4x^3, -7$

82. $-(7 \times 7 \times 7 \times 7) = -(7)^4 = -7^4$

83. $\sqrt{18x} - \sqrt{2x} = 3\sqrt{2x} - \sqrt{2x} = 2\sqrt{2x}$

84. $\sqrt[4]{x^5} = \sqrt[4]{x \cdot x^4} = |x|\sqrt[4]{x}$

85. $\dfrac{70}{\sqrt{7x}} = \dfrac{70}{\sqrt{7x}} \cdot \dfrac{\sqrt{7x}}{\sqrt{7x}} = \dfrac{70\sqrt{7x}}{7x} = \dfrac{10\sqrt{7x}}{x}$

86. $\dfrac{55}{\sqrt{20} - 3} = \dfrac{55}{\sqrt{20} - 3} \cdot \dfrac{\sqrt{20} + 3}{\sqrt{20} + 3}$

$= \dfrac{55(\sqrt{20} + 3)}{20 - 9} = \dfrac{55(\sqrt{20} + 3)}{11}$

$= 5(\sqrt{20} + 3) = 5(2\sqrt{5} + 3)$

87. $\sqrt[6]{t^2} = t^{2/6} = |t|^{1/3} = \sqrt[3]{|t|}$

88. $\sqrt[3]{\sqrt{y}} = (y^{1/2})^{1/3} = y^{1/6} = \sqrt[6]{y}$

Section 1.3 Linear Equations in Two Variables

You should know the following important facts about lines.

- The graph of $y = mx + b$ is a straight line. It is called a linear equation in two variables.
  (a) The slope (steepness) is m.
  (b) The y-intercept is $(0, b)$.
- The slope of the line through (x_1, y_1) and (x_2, y_2) is
  $$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise}}{\text{run}}.$$
- (a) If $m > 0$, the line rises from left to right.
  (b) If $m = 0$, the line is horizontal.
  (c) If $m < 0$, the line falls from left to right.
  (d) If m is undefined, the line is vertical.
- Equations of Lines
  (a) Slope-Intercept Form: $y = mx + b$
  (b) Point-Slope Form: $y - y_1 = m(x - x_1)$
  (c) Two-Point Form: $y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
  (d) General Form: $Ax + By + C = 0$
  (e) Vertical Line: $x = a$
  (f) Horizontal Line: $y = b$
- Given two distinct nonvertical lines
  L_1: $y = m_1x + b_1$ and L_2: $y = m_2x + b_2$
  (a) L_1 is parallel to L_2 if and only if $m_1 = m_2$ and $b_1 \neq b_2$.
  (b) L_1 is perpendicular to L_2 if and only if $m_1 = -1/m_2$.

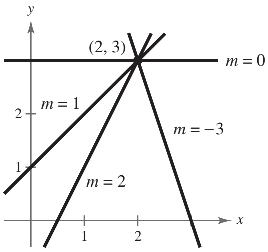
Vocabulary Check

1. linear
2. slope
3. parallel
4. perpendicular
5. rate or rate of change
6. linear extrapolation
7. (a) $Ax + By + C = 0$ (iii) general form
   (b) $x = a$ (i) vertical line
   (c) $y = b$ (v) horizontal line
   (d) $y = mx + b$ (ii) slope-intercept form
   (e) $y - y_1 = m(x - x_1)$ (iv) point-slope form

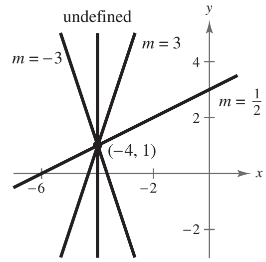
1. (a) $m = \frac{2}{3}$. Since the slope is positive, the line rises. Matches L_2.
(b) m is undefined. The line is vertical. Matches L_3.
(c) $m = -2$. The line falls. Matches L_1.

2. (a) $m = 0$. The line is horizontal. Matches L_2.
(b) $m = -\frac{3}{4}$. Because the slope is negative, the line falls. Matches L_1.
(c) $m = 1$. Because the slope is positive, the line rises. Matches L_3.

3.



4.



5. Two points on the line: $(0, 0)$ and $(4, 6)$

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{6}{4} = \frac{3}{2}$$

6. The line appears to go through $(1, 0)$ and $(3, 5)$.

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 0}{3 - 1} = \frac{5}{2}$$

7. Two points on the line: $(0, 8)$ and $(2, 0)$

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{-8}{2} = -4$$

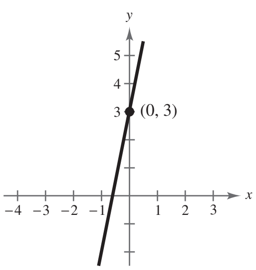
8. The line appears to go through $(0, 7)$ and $(7, 0)$.

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 7}{7 - 0} = -1$$

9. $y = 5x + 3$

Slope: $m = 5$

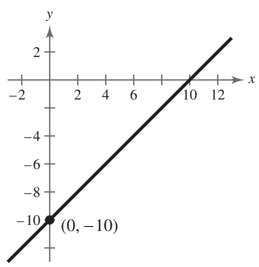
y-intercept: $(0, 3)$



10. $y = x - 10$

Slope: $m = 1$

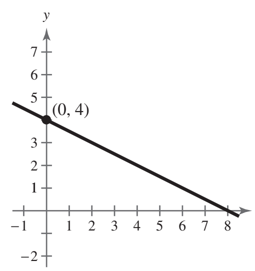
y-intercept: $(0, -10)$



11. $y = -\frac{1}{2}x + 4$

Slope: $m = -\frac{1}{2}$

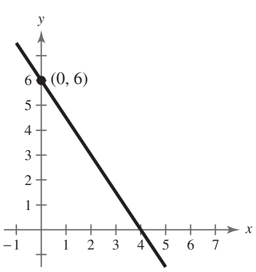
y-intercept: $(0, 4)$



12. $y = -\frac{3}{2}x + 6$

Slope: $m = -\frac{3}{2}$

y-intercept: $(0, 6)$

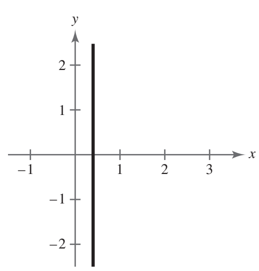


13. $5x - 2 = 0$

$x = \frac{2}{5}$, vertical line

Slope: undefined

No y-intercept



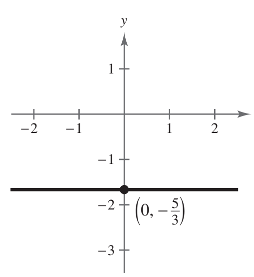
14. $3y + 5 = 0$

$3y = -5$

$y = -\frac{5}{3}$

Slope: $m = 0$

y-intercept: $\left(0, -\frac{5}{3}\right)$

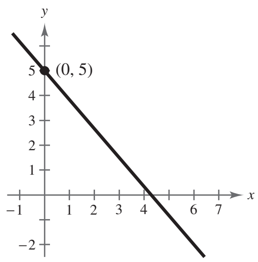


15. $7x + 6y = 30$

$y = -\frac{7}{6}x + 5$

Slope: $m = -\frac{7}{6}$

y-intercept: $(0, 5)$



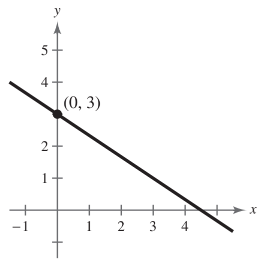
16. $2x + 3y = 9$

$3y = -2x + 9$

$y = -\frac{2}{3}x + 3$

Slope: $m = -\frac{2}{3}$

y-intercept: $(0, 3)$

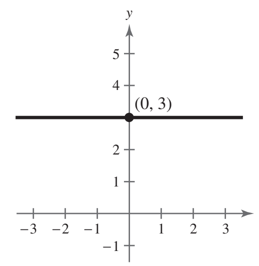


17. $y - 3 = 0$

$y = 3$, horizontal line

Slope: $m = 0$

y-intercept: $(0, 3)$

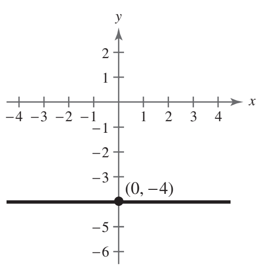


18. $y + 4 = 0$

$y = -4$

Slope: $m = 0$

y-intercept: $(0, -4)$

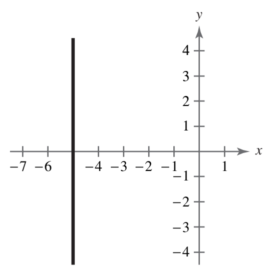


19. $x + 5 = 0$

$x = -5$

Slope: undefined (vertical line)

No y-intercept

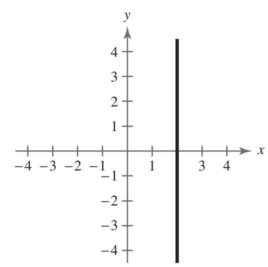


20. $x - 2 = 0$

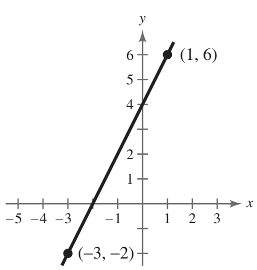
$x = 2$

Slope: undefined (vertical line)

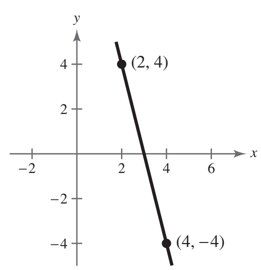
y-intercept: none



21. $m = \dfrac{6 - (-2)}{1 - (-3)} = \dfrac{8}{4} = 2$

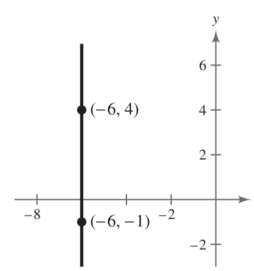


22. Slope $= \dfrac{-4 - 4}{4 - 2} = -4$

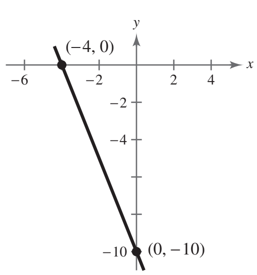


23. $m = \dfrac{4 - (-1)}{-6 - (-6)} = \dfrac{5}{0}$

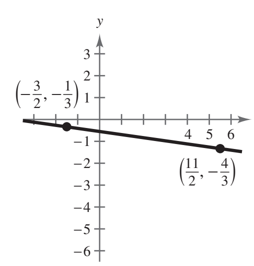
m is undefined.



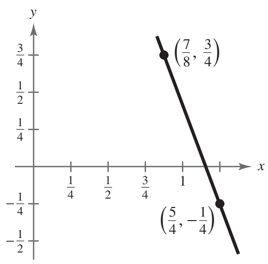
24. Slope $= \dfrac{0 - (-10)}{-4 - 0} = -\dfrac{5}{2}$



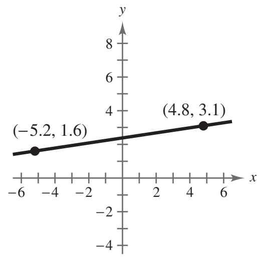
25. $m = \dfrac{-\frac{1}{3} - \left(-\frac{4}{3}\right)}{-\frac{3}{2} - \frac{11}{2}} = -\dfrac{1}{7}$



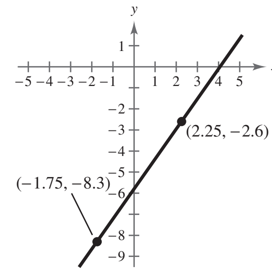
26. Slope $= \dfrac{-\frac{1}{4} - \frac{3}{4}}{\frac{5}{4} - \frac{7}{8}} = \dfrac{-1}{\frac{3}{8}} = -\dfrac{8}{3}$



27. $m = \dfrac{1.6 - 3.1}{-5.2 - 4.8} = \dfrac{-1.5}{-10} = 0.15$



28. Slope $= \dfrac{-2.6 - (-8.3)}{2.25 - (-1.75)} = 1.425$



29. Point: $(2, 1)$, Slope: $m = 0$

Since $m = 0$, y does not change. Three points are $(0, 1)$, $(3, 1)$, and $(-1, 1)$.

30. Point: $(-4, 1)$, Slope is undefined.

Because m is undefined, x does not change. Three other points are: $(-4, 0)$, $(-4, 3)$, $(-4, 5)$.

31. Point: $(5, -6)$, Slope: $m = 1$

Since $m = 1$, y increases by 1 for every one unit increase in x. Three points are $(6, -5)$, $(7, -4)$, and $(8, -3)$.

32. Point: $(10, -6)$, Slope: $m = -1$

Because $m = -1$, y decreases by 1 for every one unit increase in x. Three other points are: $(0, 4)$, $(9, -5)$, $(11, -7)$.

33. Point: $(-8, 1)$, Slope is undefined.

Since m is undefined, x does not change. Three points are $(-8, 0)$, $(-8, 2)$, and $(-8, 3)$.

34. Point: $(-3, -1)$, Slope: $m = 0$

Because $m = 0$, y does not change. Three other points are: $(-4, -1)$, $(-2, -1)$, $(0, -1)$.

35. Point: $(-5, 4)$, Slope: $m = 2$

Since $m = 2 = \frac{2}{1}$, y increases by 2 for every one unit increase in x. Three additional points are $(-4, 6)$, $(-3, 8)$, and $(-2, 10)$.

36. Point: $(0, -9)$, Slope: $m = -2$

Because $m = -2$, y decreases by 2 for every one unit increase in x. Three other points are: $(-2, -5)$, $(1, -11)$, $(3, -15)$.

37. Point: $(7, -2)$, Slope: $m = \frac{1}{2}$

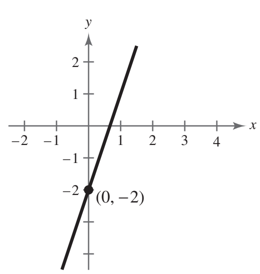
Since $m = \frac{1}{2}$, y increases by 1 unit for every two unit increase in x. Three additional points are $(9, -1)$, $(11, 0)$, and $(13, 1)$.

38. Point: $(-1, -6)$, Slope: $m = -\frac{1}{2}$

Because $m = -\frac{1}{2}$, y decreases by 1 for every 2 unit increase in x. Three other points are: $(-3, -5)$, $(1, -7)$, $(5, -9)$.

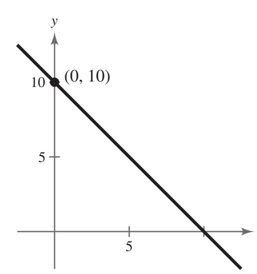
39. Point $(0, -2)$; $m = 3$

$$y + 2 = 3(x - 0)$$
$$y = 3x - 2$$



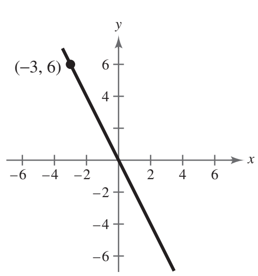
40. Point $(0, 10)$; $m = -1$

$$y - 10 = -1(x - 0)$$
$$y - 10 = -x$$
$$y = -x + 10$$



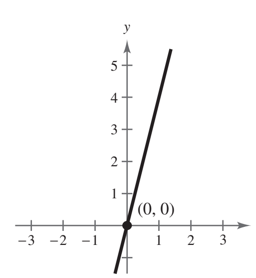
41. Point $(-3, 6)$; $m = -2$

$$y - 6 = -2(x + 3)$$
$$y = -2x$$



42. Point $(0, 0)$; $m = 4$

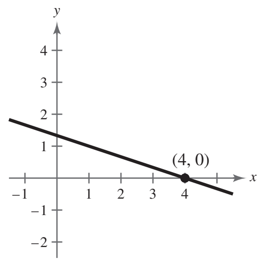
$$y - 0 = 4(x - 0)$$
$$y = 4x$$



43. Point $(4, 0)$; $m = -\frac{1}{3}$

$y - 0 = -\frac{1}{3}(x - 4)$

$y = -\frac{1}{3}x + \frac{4}{3}$



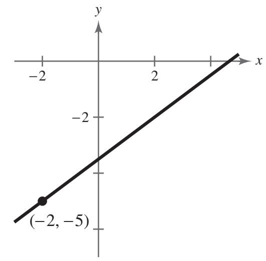
44. Point $(-2, -5)$; $m = \frac{3}{4}$

$y + 5 = \frac{3}{4}(x + 2)$

$4y + 20 = 3x + 6$

$4y = 3x - 14$

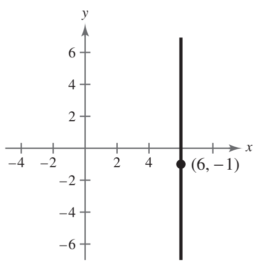
$y = \frac{3}{4}x - \frac{7}{2}$



45. Point $(6, -1)$; m is undefined.

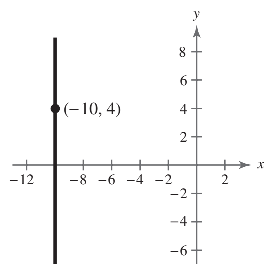
The line is vertical.

$x = 6$



46. Point $(-10, 4)$; m is undefined.

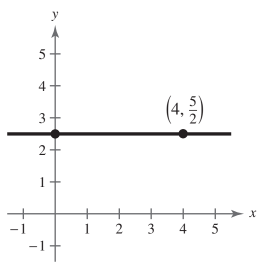
Because the slope is undefined, the line is a vertical line passing through $x = -10$, which is the equation.



47. Point $\left(4, \frac{5}{2}\right)$; $m = 0$

The line is horizontal.

$y = \frac{5}{2}$

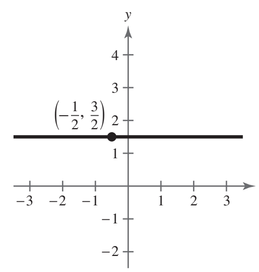


48. Point $\left(-\frac{1}{2}, \frac{3}{2}\right)$; $m = 0$

$y - \frac{3}{2} = 0\left(x + \frac{1}{2}\right)$

$y - \frac{3}{2} = 0$

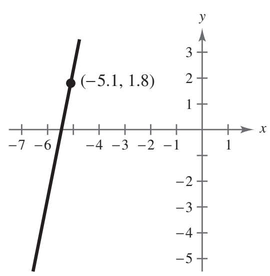
$y = \frac{3}{2}$



49. Point $(-5.1, 1.8)$; $m = 5$

$y - 1.8 = 5(x - (-5.1))$

$y = 5x + 27.3$

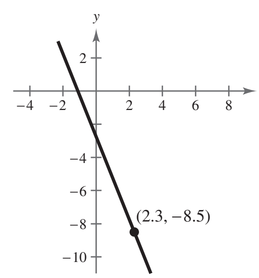


50. Point $(2.3, -8.5)$; $m = -\frac{5}{2}$

$y - (-8.5) = -\frac{5}{2}(x - 2.3)$

$y + 8.5 = -2.5x + 5.75$

$y = -2.5x - 2.75$

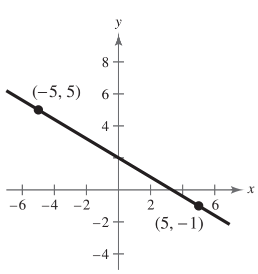


51. $(5, -1)$ and $(-5, 5)$

$$y + 1 = \frac{5 + 1}{-5 - 5}(x - 5)$$

$$y = -\frac{3}{5}(x - 5) - 1$$

$$y = -\frac{3}{5}x + 2$$



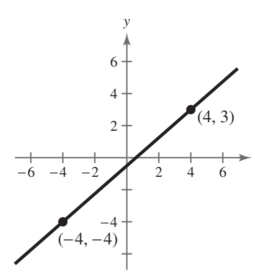
52. $(4, 3)$, $(-4, -4)$

$$y - 3 = \frac{-4 - 3}{-4 - 4}(x - 4)$$

$$y - 3 = \frac{7}{8}(x - 4)$$

$$y - 3 = \frac{7}{8}x - \frac{7}{2}$$

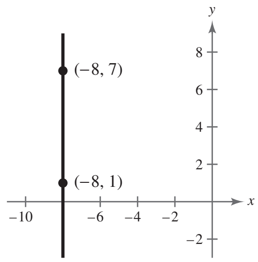
$$y = \frac{7}{8}x - \frac{1}{2}$$



53. $(-8, 1)$ and $(-8, 7)$

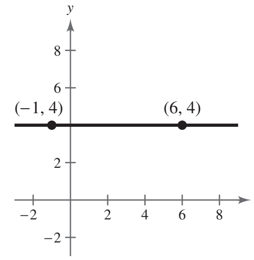
Since both points have $x = -8$, the slope is undefined, and the line is vertical.

$x = -8$



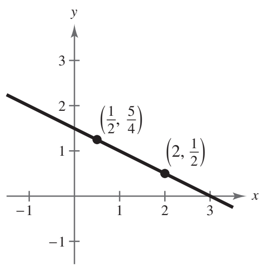
54. $(-1, 4), (6, 4)$

$$y - 4 = \frac{4 - 4}{6 - (-1)}(x + 1)$$
$$y - 4 = 0(x + 1)$$
$$y - 4 = 0$$
$$y = 4$$



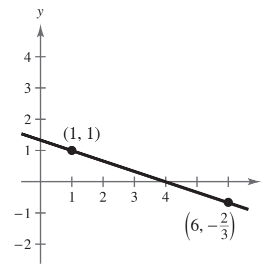
55. $\left(2, \frac{1}{2}\right)$ and $\left(\frac{1}{2}, \frac{5}{4}\right)$

$$y - \frac{1}{2} = \frac{\frac{5}{4} - \frac{1}{2}}{\frac{1}{2} - 2}(x - 2)$$
$$y = -\frac{1}{2}(x - 2) + \frac{1}{2}$$
$$y = -\frac{1}{2}x + \frac{3}{2}$$



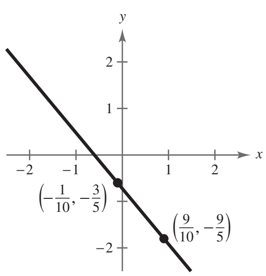
56. $(1, 1), \left(6, -\frac{2}{3}\right)$

$$y - 1 = \frac{-\frac{2}{3} - 1}{6 - 1}(x - 1)$$
$$y - 1 = -\frac{1}{3}(x - 1)$$
$$y - 1 = -\frac{1}{3}x + \frac{1}{3}$$
$$y = -\frac{1}{3}x + \frac{4}{3}$$



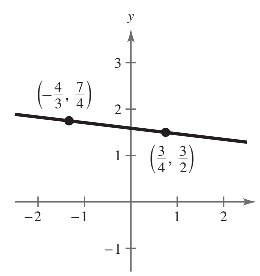
57. $\left(-\frac{1}{10}, -\frac{3}{5}\right)$ and $\left(\frac{9}{10}, -\frac{9}{5}\right)$

$$y - \left(-\frac{3}{5}\right) = \frac{-\frac{9}{5} - \left(-\frac{3}{5}\right)}{\frac{9}{10} - \left(-\frac{1}{10}\right)}\left(x - \left(-\frac{1}{10}\right)\right)$$
$$y = -\frac{6}{5}\left(x + \frac{1}{10}\right) - \frac{3}{5}$$
$$y = -\frac{6}{5}x - \frac{18}{25}$$



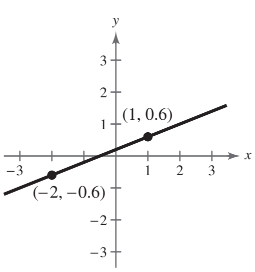
58. $\left(\frac{3}{4}, \frac{3}{2}\right), \left(-\frac{4}{3}, \frac{7}{4}\right)$

$$y - \frac{3}{2} = \frac{\frac{7}{4} - \frac{3}{2}}{-\frac{4}{3} - \frac{3}{4}}\left(x - \frac{3}{4}\right)$$
$$y - \frac{3}{2} = \frac{\frac{1}{4}}{-\frac{25}{12}}\left(x - \frac{3}{4}\right)$$
$$y - \frac{3}{2} = -\frac{3}{25}\left(x - \frac{3}{4}\right)$$
$$y - \frac{3}{2} = -\frac{3}{25}x + \frac{9}{100}$$
$$y = -\frac{3}{25}x + \frac{159}{100}$$



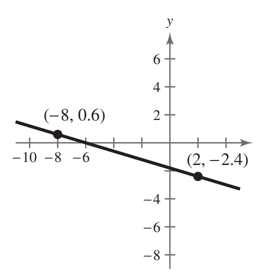
59. $(1, 0.6)$ and $(-2, -0.6)$

$$y - 0.6 = \frac{-0.6 - 0.6}{-2 - 1}(x - 1)$$
$$y = 0.4(x - 1) + 0.6$$
$$y = 0.4x + 0.2$$



60. $(-8, 0.6), (2, -2.4)$

$$y - 0.6 = \frac{-2.4 - 0.6}{2 - (-8)}(x + 8)$$
$$y - 0.6 = -\frac{3}{10}(x + 8)$$
$$10y - 6 = -3(x + 8)$$
$$10y - 6 = -3x - 24$$
$$10y = -3x - 18$$
$$y = -\frac{3}{10}x - \frac{9}{5} \quad \text{or} \quad y = -0.3x - 1.8$$



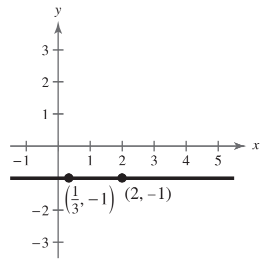
61. $(2, -1)$ and $\left(\frac{1}{3}, -1\right)$

$$y + 1 = \frac{-1 - (-1)}{\frac{1}{3} - 2}(x - 2)$$

$$y + 1 = 0$$

$$y = -1$$

The line is horizontal.



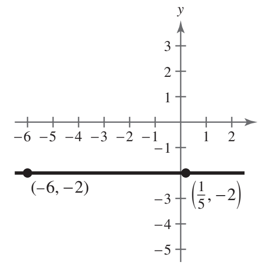
62. $\left(\frac{1}{5}, -2\right), (-6, -2)$

$$y + 2 = \frac{-2 - (-2)}{-6 - \frac{1}{5}}(x + 6)$$

$$y + 2 = \frac{0}{-6 - \frac{1}{5}}(x + 6)$$

$$y + 2 = 0$$

$$y = -2$$

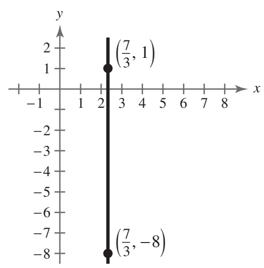


63. $\left(\frac{7}{3}, -8\right)$ and $\left(\frac{7}{3}, 1\right)$

$m = \dfrac{1 - (-8)}{\frac{7}{3} - \frac{7}{3}} = \dfrac{9}{0}$ and is undefined.

$$x = \frac{7}{3}$$

The line is vertical.



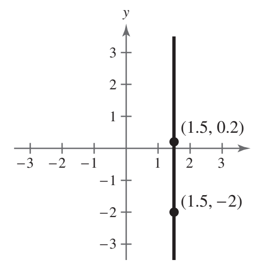
64. $(1.5, -2), (1.5, 0.2)$

$$y + 2 = \frac{-2 - 0.2}{1.5 - 1.5}(x - 1.5)$$

$$y + 2 = \frac{-2 - 0.2}{0}(x - 1.5)$$

The slope is undefined. The line is vertical.

$$x = 1.5$$



65. L_1: $(0, -1), (5, 9)$

Slope of L_1: $m = \dfrac{9 + 1}{5 - 0} = 2$

L_2: $(0, 3), (4, 1)$

Slope of L_2: $m = \dfrac{1 - 3}{4 - 0} = -\dfrac{1}{2}$

L_1 and L_2 are perpendicular.

66. L_1: $(-2, -1), (1, 5)$

$$m_1 = \frac{5 - (-1)}{1 - (-2)} = \frac{6}{3} = 2$$

L_2: $(1, 3), (5, -5)$

$$m_2 = \frac{-5 - 3}{5 - 1} = \frac{-8}{4} = -2$$

The lines are neither parallel nor perpendicular.

67. L_1: $(3, 6), (-6, 0)$

Slope of L_1: $m = \dfrac{0 - 6}{-6 - 3} = \dfrac{2}{3}$

L_2: $(0, -1), \left(5, \frac{7}{3}\right)$

Slope of L_2: $m = \dfrac{\frac{7}{3} + 1}{5 - 0} = \dfrac{2}{3}$

L_1 and L_2 are parallel.

68. L_1: $(4, 8), (-4, 2)$

$$m_1 = \frac{2 - 8}{-4 - 4} = \frac{-6}{-8} = \frac{3}{4}$$

L_2: $(3, -5), \left(-1, \frac{1}{3}\right)$

$$m_2 = \frac{\frac{1}{3} - (-5)}{-1 - 3} = \frac{\frac{16}{3}}{-4} = -\frac{4}{3}$$

The lines are perpendicular.

69. $4x - 2y = 3$

$$y = 2x - \frac{3}{2}$$

Slope: $m = 2$

(a) $(2, 1), m = 2$

$$y - 1 = 2(x - 2)$$

$$y = 2x - 3$$

(b) $(2, 1), m = -\dfrac{1}{2}$

$$y - 1 = -\frac{1}{2}(x - 2)$$

$$y = -\frac{1}{2}x + 2$$

70. $x + y = 7$

$$y = -x + 7$$

Slope: $m = -1$

(a) $m = -1, (-3, 2)$

$$y - 2 = -1(x + 3)$$

$$y - 2 = -x - 3$$

$$y = -x - 1$$

(b) $m = 1, (-3, 2)$

$$y - 2 = 1(x + 3)$$

$$y = x + 5$$

71. $3x + 4y = 7$

$$y = -\frac{3}{4}x + \frac{7}{4}$$

Slope: $m = -\frac{3}{4}$

(a) $\left(-\frac{2}{3}, \frac{7}{8}\right), m = -\frac{3}{4}$

$$y - \frac{7}{8} = -\frac{3}{4}\left(x - \left(-\frac{2}{3}\right)\right)$$

$$y = -\frac{3}{4}x + \frac{3}{8}$$

(b) $\left(-\frac{2}{3}, \frac{7}{8}\right), m = \frac{4}{3}$

$$y - \frac{7}{8} = \frac{4}{3}\left(x - \left(-\frac{2}{3}\right)\right)$$

$$y = \frac{4}{3}x + \frac{127}{72}$$

72. $5x + 3y = 0$

$$3y = -5x$$

$$y = -\frac{5}{3}x$$

Slope: $m = -\frac{5}{3}$

(a) $m = -\frac{5}{3}, \left(\frac{7}{8}, \frac{3}{4}\right)$

$$y - \frac{3}{4} = -\frac{5}{3}\left(x - \frac{7}{8}\right)$$

$$24y - 18 = -40\left(x - \frac{7}{8}\right)$$

$$24y - 18 = -40x + 35$$

$$24y = -40x + 53$$

$$y = -\frac{5}{3}x + \frac{53}{24}$$

(b) $m = \frac{3}{5}, \left(\frac{7}{8}, \frac{3}{4}\right)$

$$y - \frac{3}{4} = \frac{3}{5}\left(x - \frac{7}{8}\right)$$

$$40y - 30 = 24\left(x - \frac{7}{8}\right)$$

$$40y - 30 = 24x - 21$$

$$40y = 24x + 9$$

$$y = \frac{3}{5}x + \frac{9}{40}$$

73. $y = -3$

$m = 0$

(a) $(-1, 0)$ and $m = 0$

$$y = 0$$

(b) $(-1, 0)$, m is undefined.

$$x = -1$$

74. $y = 1$

Slope: $m = 0$

(a) $m = 0, (4, -2)$

$$y + 2 = 0(x - 4)$$

$$y + 2 = 0$$

$$y = -2$$

(b) The reciprocal of 0 is undefined. The line is vertical, passing through $(4, -2)$.

$$x = 4$$

75. $x = 4$

m is undefined.

(a) $(2, 5)$, m is undefined. The line is vertical, passing through $(2, 5)$.

$$x = 2$$

(b) $(2, 5), m = 0$

$$y = 5$$

76. $x = -2$

Slope: undefined

(a) The original line is the vertical line through $x = -2$. The line parallel to this line containing $(-5, 1)$ is the vertical line $x = -5$.

(b) A perpendicular to a vertical line is a horizontal line, whose slope is 0. The horizontal line containing $(-5, 1)$ is the line $y = 1$.

77. $x - y = 4$

$$y = x - 4$$

Slope: $m = 1$

(a) $(2.5, 6.8), m = 1$

$$y - 6.8 = 1(x - 2.5)$$

$$y = x + 4.3$$

(b) $(2.5, 6.8), m = -1$

$$y - 6.8 = (-1)(x - 2.5)$$

$$y = -x + 9.3$$

78. $6x + 2y = 9$

$$2y = -6x + 9$$

$$y = -3x + \frac{9}{2}$$

Slope: $m = -3$

(a) $(-3.9, -1.4), m = -3$

$$y - (-1.4) = -3(x - (-3.9))$$

$$y + 1.4 = -3x - 11.7$$

$$y = -3x - 13.1$$

(b) $(-3.9, -1.4), m = \frac{1}{3}$

$$y - (-1.4) = \frac{1}{3}(x - (-3.9))$$

$$y + 1.4 = \frac{1}{3}x + 1.3$$

$$y = \frac{1}{3}x - 0.1$$

79. $\dfrac{x}{2} + \dfrac{y}{3} = 1$

$3x + 2y - 6 = 0$

80. $(-3, 0), (0, 4)$

$$\frac{x}{-3} + \frac{y}{4} = 1$$

$$(-12)\frac{x}{-3} + (-12)\frac{y}{4} = (-12) \cdot 1$$

$$4x - 3y + 12 = 0$$

81. $\dfrac{x}{-1/6} + \dfrac{y}{-2/3} = 1$

$$6x + \frac{3}{2}y = -1$$

$$12x + 3y + 2 = 0$$

82. $\left(\dfrac{2}{3}, 0\right), (0, -2)$

$$\frac{x}{2/3} + \frac{y}{-2} = 1$$

$$\frac{3x}{2} - \frac{y}{2} = 1$$

$$3x - y - 2 = 0$$

83. $\dfrac{x}{c} + \dfrac{y}{c} = 1,\ c \neq 0$

$$x + y = c$$

$$1 + 2 = c$$

$$3 = c$$

$$x + y = 3$$

$$x + y - 3 = 0$$

84. $(d, 0), (0, d), (-3, 4)$

$$\frac{x}{d} + \frac{y}{d} = 1$$

$$x + y = d$$

$$-3 + 4 = d$$

$$1 = d$$

$$x + y = 1$$

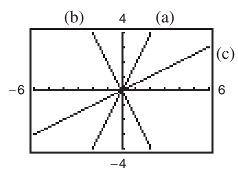
$$x + y - 1 = 0$$

85. (a) $y = 2x$

(b) $y = -2x$

(c) $y = \frac{1}{2}x$

(b) and (c) are perpendicular.

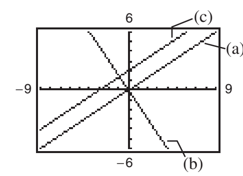


86. (a) $y = \frac{2}{3}x$

(b) $y = -\frac{3}{2}x$

(c) $y = \frac{2}{3}x + 2$

(a) is parallel to (c). (b) is perpendicular to (a) and (c).

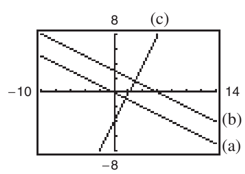


87. (a) $y = -\frac{1}{2}x$

(b) $y = -\frac{1}{2}x + 3$

(c) $y = 2x - 4$

(a) and (b) are parallel. (c) is perpendicular to (a) and (b).

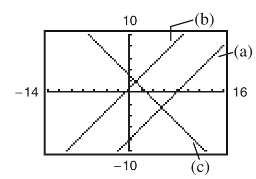


88. (a) $y = x - 8$

(b) $y = x + 1$

(c) $y = -x + 3$

(a) is parallel to (b). (c) is perpendicular to (a) and (b).



89. Set the distance between $(4, -1)$ and (x, y) equal to the distance between $(-2, 3)$ and (x, y).

$$\sqrt{(x-4)^2 + [y-(-1)]^2} = \sqrt{[x-(-2)]^2 + (y-3)^2}$$

$$(x-4)^2 + (y+1)^2 = (x+2)^2 + (y-3)^2$$

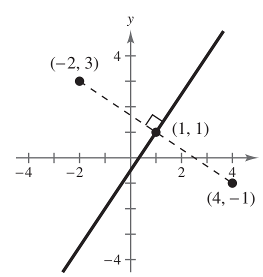
$$x^2 - 8x + 16 + y^2 + 2y + 1 = x^2 + 4x + 4 + y^2 - 6y + 9$$

$$-8x + 2y + 17 = 4x - 6y + 13$$

$$0 = 12x - 8y - 4$$

$$0 = 4(3x - 2y - 1)$$

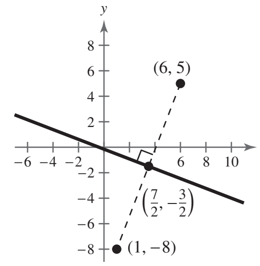
$$0 = 3x - 2y - 1$$



This line is the perpendicular bisector of the line segment connecting $(4, -1)$ and $(-2, 3)$.

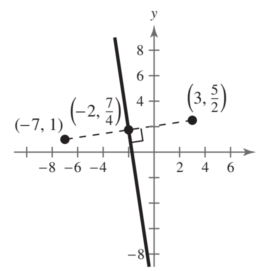
90. Set the distance between $(6, 5)$ and (x, y) equal to the distance between $(1, -8)$ and (x, y).

$$\sqrt{(x-6)^2 + (y-5)^2} = \sqrt{(x-1)^2 + (y-(-8))^2}$$
$$(x-6)^2 + (y-5)^2 = (x-1)^2 + (y+8)^2$$
$$x^2 - 12x + 36 + y^2 - 10y + 25 = x^2 - 2x + 1 + y^2 + 16y + 64$$
$$x^2 + y^2 - 12x - 10y + 61 = x^2 + y^2 - 2x + 16y + 65$$
$$-12x - 10y + 61 = -2x + 16y + 65$$
$$-10x - 26y - 4 = 0$$
$$-2(5x + 13y + 2) = 0$$
$$5x + 13y + 2 = 0$$



91. Set the distance between $\left(3, \frac{5}{2}\right)$ and (x, y) equal to the distance between $(-7, 1)$ and (x, y).

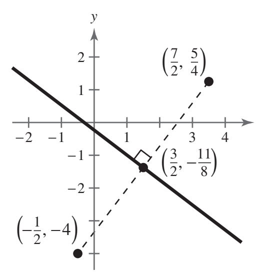
$$\sqrt{(x-3)^2 + \left(y - \frac{5}{2}\right)^2} = \sqrt{[x-(-7)]^2 + (y-1)^2}$$
$$(x-3)^2 + \left(y - \frac{5}{2}\right)^2 = (x+7)^2 + (y-1)^2$$
$$x^2 - 6x + 9 + y^2 - 5y + \frac{25}{4} = x^2 + 14x + 49 + y^2 - 2y + 1$$
$$-6x - 5y + \frac{61}{4} = 14x - 2y + 50$$
$$-24x - 20y + 61 = 56x - 8y + 200$$
$$80x + 12y + 139 = 0$$



This line is the perpendicular bisector of the line segment connecting $\left(3, \frac{5}{2}\right)$ and $(-7, 1)$.

92. Set the distance between $\left(-\frac{1}{2}, -4\right)$ and (x, y) equal to the distance between $\left(\frac{7}{2}, \frac{5}{4}\right)$ and (x, y).

$$\sqrt{\left(x - \left(-\frac{1}{2}\right)\right)^2 + (y-(-4))^2} = \sqrt{\left(x - \frac{7}{2}\right)^2 + \left(y - \frac{5}{4}\right)^2}$$
$$\left(x + \frac{1}{2}\right)^2 + (y+4)^2 = \left(x - \frac{7}{2}\right)^2 + \left(y - \frac{5}{4}\right)^2$$
$$x^2 + x + \frac{1}{4} + y^2 + 8y + 16 = x^2 - 7x + \frac{49}{4} + y^2 - \frac{5}{2}y + \frac{25}{16}$$
$$x^2 + y^2 + x + 8y + \frac{65}{4} = x^2 + y^2 - 7x - \frac{5}{2}y + \frac{221}{16}$$
$$x + 8y + \frac{65}{4} = -7x - \frac{5}{2}y + \frac{221}{16}$$
$$8x + \frac{21}{2}y + \frac{39}{16} = 0$$
$$128x + 168y + 39 = 0$$



93. (a) $m = 135$. The sales are increasing 135 units per year.

(b) $m = 0$. There is no change in sales during the year.

(c) $m = -40$. The sales are decreasing 40 units per year.

94. (a) $m = 400$. The revenues are increasing 400 units per day.

(b) $m = 100$. The revenues are increasing 100 units per day.

(c) $m = 0$. There is no change in revenue during the day. (Revenue remains constant.)

95. (a) $(0, 55{,}722), (2, 61{,}768)$: $m = \dfrac{61{,}768 - 55{,}722}{2 - 0} = 3023$

$(2, 61{,}768), (4, 64{,}993)$: $m = \dfrac{64{,}993 - 61{,}768}{4 - 2} = 1612.5$

$(4, 64{,}993), (6, 69{,}277)$: $m = \dfrac{69{,}277 - 64{,}993}{6 - 4} = 2142$

$(6, 69{,}277), (8, 74{,}380)$: $m = \dfrac{74{,}380 - 69{,}277}{8 - 6} = 2551.5$

$(8, 74{,}380), (10, 79{,}839)$: $m = \dfrac{79{,}839 - 74{,}380}{10 - 8} = 2729.5$

$(10, 79{,}839), (12, 83{,}944)$: $m = \dfrac{83{,}944 - 79{,}839}{12 - 10} = 2052.5$

The average salary increased the most from 1990 to 1992 and the least from 1992 to 1994.

—CONTINUED—

95. —CONTINUED—

(b) $(0, 55{,}722), (12, 83{,}944)$: $m = \dfrac{83{,}944 - 55{,}722}{12 - 0} \approx \$2351.83$

(c) The average salary for senior high school principals increased by \$2351.83 per year over the 12 years between 1990 and 2002.

96. (a) The greatest increase of \$16.2 million is between 2002 and 2003. The least increase of \$5.4 million is between 2000 and 2001.

(b) Slope $= \dfrac{99.2 - 16.6}{13 - 4} = 9.18$

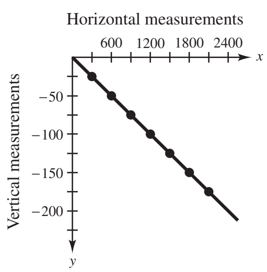
(c) Each year the net profit increases by \$9.18 million.

97. $y = \dfrac{6}{100}x$

$y = \dfrac{6}{100}(200) = 12$ feet

98. (a) and (b)

x	300	600	900	1200	1500	1800	2100
y	-25	-50	-75	-100	-125	-150	-175



(c) $m = \dfrac{-50 - (-25)}{600 - 300} = \dfrac{-25}{300} = -\dfrac{1}{12}$

$$y - (-50) = -\frac{1}{12}(x - 600)$$

$$y + 50 = -\frac{1}{12}x + 50$$

$$y = -\frac{1}{12}x$$

(d) Since $m = -\frac{1}{12}$, for every change in the horizontal measurement of 12 units, the vertical measurement decreases by 1.

(e) $\dfrac{1}{12} \approx 0.083 = 8.3\%$ grade

99. $(5, 2540)$, $m = -125$

$$V - 2540 = -125(t - 5)$$

$$V - 2540 = -125t + 625$$

$$V = -125t + 3165,\ 5 \le t \le 10$$

100. $(5, 156)$, $m = 4.50$

$$V - 156 = 4.50(t - 5)$$

$$V - 156 = 4.50t - 22.5$$

$$V = 4.5t + 133.5,\ 5 \le t \le 10$$

101. Matches graph (b).

The slope is -20, which represents the decrease in the amount of the loan each week. The y-intercept is $(0, 200)$, which represents the original amount of the loan.

102. Matches graph (c).

The slope is 2, which represents the increase in the hourly wage for each unit produced. The y-intercept is $(0, 8.5)$, which represents the hourly rate if the employee produces no units.

103. Matches graph (a).

The slope is 0.32, which represents the increase in travel cost for each mile driven. The y-intercept is $(0, 30)$, which represents the fixed cost of \$30 per day for meals. This amount does not depend on the number of miles driven.

104. Matches graph (d).

The slope is -100, which represents the amount by which the computer depreciates each year. The y-intercept is $(0, 750)$, which represents the original purchase price.

105. $(5, 0.18), (13, 4.04)$: $m = \dfrac{4.04 - 0.18}{13 - 5} = 0.4825$

$y - 0.18 = 0.4825(t - 5)$

$y = 0.4825t - 2.2325$

For 2008, use $t = 18$: $y(18) \approx \$6.45$

For 2010, use $t = 20$: $y(20) \approx \$7.42$

106. $t = 9$ represents 1999, $(9, 4076)$.

$t = 13$ represents 2003, $(13, 1078)$.

$m = \dfrac{4076 - 1078}{9 - 13} = \dfrac{-2998}{4} = -749.5$

$N = -749.5t + 10{,}821.5$

$t = 18$ represents 2008:

$N = -749.5(18) + 10{,}821.5 = -2669.5$ stores

$t = 20$ represents 2010:

$N = -749.5(20) + 10{,}821.5 = -4168.5$ stores

These answers are not reasonable because they are negative.

107. Using the points $(0, 875)$ and $(5, 0)$, where the first coordinate represents the year t and the second coordinate represents the value V, we have

$m = \dfrac{0 - 875}{5 - 0} = -175$

$V = -175t + 875,\ 0 \le t \le 5.$

108. $(0, 25{,}000)$ and $(10, 2000)$

$m = \dfrac{2000 - 25000}{10 - 0} = -2300$

$V = -2300t + 25{,}000, \quad 0 \le t \le 10$

109. (a) $(0, 40{,}571), (4, 41{,}289)$:

$m = \dfrac{41{,}289 - 40{,}571}{4 - 0} = 179.5$

$y = 179.5t + 40{,}571$

(b) For 2008, use $t = 8$: $y(8) = 42{,}007$ students.

For 2010, use $t = 10$: $y(10) = 42{,}366$ students.

(c) The slope is $m = 179.5$, which represents the increase in the number of students each year.

110. (a) Average annual salary change from 1990 to 2003:

$\dfrac{48{,}673 - 36{,}531}{13 - 0} = \dfrac{12{,}142}{13} = 934$ students per year

(b) Using (a) to estimate the enrollment in:

1994: $36{,}531 + 4(934) = 40{,}267$ students

1998: $36{,}531 + 8(934) = 44{,}003$ students

2002: $36{,}531 + 12(934) = 47{,}739$ students

(c) $m = 934$, $b = 36{,}531$, so $N(t) = 934t + 36{,}531$.

The slope, 934, represents the average annual change in enrollment.

(d) Answers will vary.

111. Sale price $=$ List price $-$ 15% of the list price

$S = L - 0.15L$

$S = 0.85L$

112. $W = 0.75x + 11.50$

113. (a) $C = 36{,}500 + 5.25t + 11.50t$

$= 16.75t + 36{,}500$

(b) $R = 27t$

(c) $P = R - C$

$= 27t - (16.75t + 36{,}500)$

$= 10.25t - 36{,}500$

(d) $0 = 10.25t - 36{,}500$

$36{,}500 = 10.25t$

$t \approx 3561$ hours

114. $(580, 50)$ and $(625, 47)$

(a) $m = \dfrac{47 - 50}{625 - 580} = \dfrac{-3}{45} = -\dfrac{1}{15}$

$$x - 50 = -\frac{1}{15}(p - 580)$$

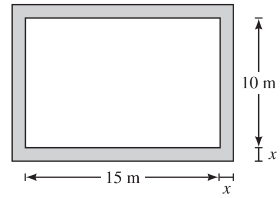
$$x - 50 = -\frac{1}{15}p + \frac{116}{3}$$

$$x = -\frac{1}{15}p + \frac{266}{3}$$

(b) $x = -\dfrac{1}{15}(655) + \dfrac{266}{3} = 45$ units

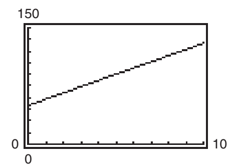
(c) $x = -\dfrac{1}{15}(595) + \dfrac{266}{3} = 49$ units

115. (a)



(b) $y = 2(15 + 2x) + 2(10 + 2x) = 8x + 50$

(c)

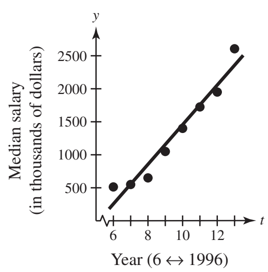


(d) Since $m = 8$, each 1-meter increase in x will increase y by 8 meters.

116. $W = 0.07S + 2500$

117. $C = 0.38x + 120$

118.



Using a calculator, the linear regression line is $y = 300.3t - 1547.4$. Choosing the points $(7, 550)$ and $(10, 1400)$:

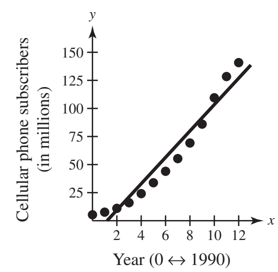
$$m = \frac{1400 - 550}{10 - 7} = \frac{850}{3} = 283.3$$

$$y - 550 = 283.3(t - 7)$$

$$y = 283.3t - 1433.1$$

The answer varies depending on the points chosen to estimate the line.

119. (a) and (b)



(c) Answers will vary. Find two points on your line and then find the equation of the line through your points. Sample answer: $y \approx 11.72x - 14.08$

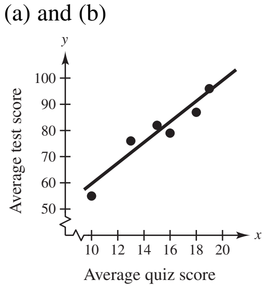
(d) Answers will vary. Sample answer: The y-intercept should represent the number of initial subscribers. In this case, since b is negative, it cannot be interpreted as such. The slope of 11.72 represents the increase in the number of subscribers per year (in millions).

(e) The model is a fairly good fit to the data.

(f) Answers will vary. Sample answer:

$$y(18) \approx 11.72(18) - 14.08$$
$$= 196.88 \text{ million subscribers in 2008}$$

120. (a) and (b)



(c) Two approximate points on the line are $(10, 55)$ and $(19, 96)$.

$$m = \frac{96 - 55}{19 - 10} = \frac{41}{9}$$

$$y - 55 = \frac{41}{9}(x - 10)$$

$$y = \frac{41}{9}x + \frac{85}{9}$$

(d) $y = \dfrac{41}{9}(17) + \dfrac{85}{9} \approx 87$

(e) Each point will shift four units upward, so the best-fitting line will move four units upward. The slope remains the same, as the new line is parallel to the old, but the y-intercept becomes

$$\left(0, \frac{85}{9} + 4\right) = \left(0, \frac{121}{9}\right)$$

so the new equation is $y = \dfrac{41}{9}x + \dfrac{121}{9}$.

121. False. The slope with the greatest magnitude corresponds to the steepest line.

122. $(-8, 2)$ and $(-1, 4): m_1 = \dfrac{4-2}{-1-(-8)} = \dfrac{2}{7}$

$(0, -4)$ and $(-7, 7): m_2 = \dfrac{7-(-4)}{-7-0} = \dfrac{11}{-7}$

False, the lines are not parallel.

123. Using the Distance Formula, we have $AB = 6$, $BC = \sqrt{40}$, and $AC = 2$. Since $6^2 + 2^2 = (\sqrt{40})^2$, the triangle is a right triangle.

124. On a vertical line, all the points have the same x-value, so when you evaluate $m = \dfrac{y_2 - y_1}{x_2 - x_1}$, you would have a zero in the denominator, and division by zero is undefined.

125. No. The slope cannot be determined without knowing the scale on the y-axis. The slopes will be the same if the scale on the y-axis of (a) is $2\frac{1}{2}$ and the scale on the y-axis of (b) is 1. Then the slope of both is $\frac{5}{4}$.

126. Since $|-4| > \left|\frac{5}{2}\right|$, the steeper line is the one with a slope of -4. The slope with the greatest magnitude corresponds to the steepest line.

127. The V-intercept measures the initial cost and the slope measures annual depreciation.

128. No, the slopes of two perpendicular lines have opposite signs. (Assume that neither line is vertical or horizontal.)

129. $y = 8 - 3x$ is a linear equation with slope $m = -3$ and y-intercept $(0, 8)$. Matches graph (d).

130. $y = 8 - \sqrt{x}$

Intercepts: $(64, 0), (0, 8)$

Matches graph (c).

131. $y = \frac{1}{2}x^2 + 2x + 1$ is a quadratic equation. Its graph is a parabola with vertex $(-2, -1)$ and y-intercept $(0, 1)$. Matches graph (a).

132. $y = |x + 2| - 1$

Intercepts: $(-1, 0), (-3, 0), (0, 1)$

Matches graph (b).

133.
$$\begin{aligned} -7(3 - x) &= 14(x - 1) \\ -21 + 7x &= 14x - 14 \\ -7x &= 7 \\ x &= -1 \end{aligned}$$

134.
$$\begin{aligned} \frac{8}{2x - 7} &= \frac{4}{9 - 4x} \\ 8(9 - 4x) &= 4(2x - 7) \\ 72 - 32x &= 8x - 28 \\ -40x &= -100 \\ x &= \frac{5}{2} \end{aligned}$$

135.
$$\begin{aligned} &2x^2 - 21x + 49 = 0 \\ &(2x - 7)(x - 7) = 0 \\ &2x - 7 = 0 \quad \text{or} \quad x - 7 = 0 \\ &x = \frac{7}{2} \quad \text{or} \quad x = 7 \end{aligned}$$

136.
$$\begin{aligned} x^2 - 8x + 3 &= 0 \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(3)}}{2(1)} \\ &= \frac{8 \pm \sqrt{52}}{2} = 4 \pm \sqrt{13} \end{aligned}$$

137.
$$\begin{aligned} \sqrt{x - 9} + 15 &= 0 \\ \sqrt{x - 9} &= -15 \end{aligned}$$

No real solution

The square root of $x - 9$ cannot be negative.

138.
$$\begin{aligned} 3x - 16\sqrt{x} + 5 &= 0 \\ \left(3\sqrt{x} - 1\right)\left(\sqrt{x} - 5\right) &= 0 \\ 3\sqrt{x} - 1 = 0 &\Rightarrow x = \tfrac{1}{9} \\ \sqrt{x} - 5 = 0 &\Rightarrow x = 25 \end{aligned}$$

139. Answers will vary.

Section 1.4 Functions

- Given a set or an equation, you should be able to determine if it represents a function.
- Know that functions can be represented in four ways: verbally, numerically, graphically, and algebraically.
- Given a function, you should be able to do the following.
  (a) Find the domain and range.
  (b) Evaluate it at specific values.
- You should be able to use function notation.

Vocabulary Check

1. domain; range; function

2. verbally; numerically; graphically; algebraically

3. independent; dependent

4. piecewise-defined

5. implied domain

6. difference quotient

1. Yes, the relationship is a function. Each domain value is matched with only one range value.

2. No, it is not a function. The domain value of -1 is matched with two output values.

3. No, the relationship is not a function. The domain values are each matched with three range values.

4. Yes, it is a function. Each domain value is matched with only one range value.

5. Yes, it does represent a function. Each input value is matched with only one output value.

6. No, the table does not represent a function. The input values of 0 and 1 are each matched with two different output values.

7. No, it does not represent a function. The input values of 10 and 7 are each matched with two output values.

8. Yes, the table does represent a function. Each input value is matched with only one output value.

9. (a) Each element of A is matched with exactly one element of B, so it does represent a function.

(b) The element 1 in A is matched with two elements, -2 and 1 of B, so it does not represent a function.

(c) Each element of A is matched with exactly one element of B, so it does represent a function.

(d) The element 2 in A is not matched with an element of B, so the relation does not represent a function.

10. (a) The element c in A is matched with two elements, 2 and 3 of B, so it is not a function.

(b) Each element of A is matched with exactly one element of B, so it does represent a function.

(c) This is not a function from A to B (it represents a function from B to A instead).

(d) Each element of A is matched with exactly one element of B, so it does represent a function.

11. Each is a function. For each year there corresponds one and only one circulation.

12. Reading from the graph, $f(1998)$ is approximately 11 million.

13. $x^2 + y^2 = 4 \implies y = \pm\sqrt{4 - x^2}$

No, y *is not* a function of x.

14. $x = y^2 \implies y = \pm\sqrt{x}$

Thus, y *is not* a function of x.

15. $x^2 + y = 4 \implies y = 4 - x^2$

Yes, y *is* a function of x.

16. $x + y^2 = 4 \implies y = \pm\sqrt{4 - x}$

Thus, y *is not* a function of x.

17. $2x + 3y = 4 \implies y = \frac{1}{3}(4 - 2x)$

Yes, y *is* a function of x.

18. $(x - 2)^2 + y^2 = 4 \implies y = \pm\sqrt{4 - (x - 2)^2}$

Thus, y *is not* a function of x.

19. $y^2 = x^2 - 1 \implies y = \pm\sqrt{x^2 - 1}$

Thus, y *is not* a function of x.

20. $y = \sqrt{x + 5}$

Yes, y *is* a function of x.

21. $y = |4 - x|$

Yes, y *is* a function of x.

22. $|y| = 4 - x \implies y = 4 - x$ or $y = -(4 - x)$

Thus, y *is not* a function of x.

23. $x = 14$

Thus, this *is not* a function of x.

24. $y = -75$ or $y = -75 + 0x$

y *is* a function of x.

25. $f(x) = 2x - 3$

(a) $f(1) = 2(1) - 3 = -1$

(b) $f(-3) = 2(-3) - 3 = -9$

(c) $f(x - 1) = 2(x - 1) - 3 = 2x - 5$

26. $g(y) = 7 - 3y$

(a) $g(0) = 7 - 3(0) = 7$

(b) $g\left(\frac{7}{3}\right) = 7 - 3\left(\frac{7}{3}\right) = 0$

(c) $g(s + 2) = 7 - 3(s + 2)$

$= 7 - 3s - 6 = 1 - 3s$

27. $V(r) = \frac{4}{3}\pi r^3$

(a) $V(3) = \frac{4}{3}\pi(3)^3 = \frac{4}{3}\pi(27) = 36\pi$

(b) $V\left(\frac{3}{2}\right) = \frac{4}{3}\pi\left(\frac{3}{2}\right)^3 = \frac{4}{3}\pi\left(\frac{27}{8}\right) = \frac{9}{2}\pi$

(c) $V(2r) = \frac{4}{3}\pi(2r)^3 = \frac{4}{3}\pi(8r^3) = \frac{32}{3}\pi r^3$

28. $h(t) = t^2 - 2t$

(a) $h(2) = 2^2 - 2(2) = 0$

(b) $h(1.5) = (1.5)^2 - 2(1.5) = -0.75$

(c) $h(x + 2) = (x + 2)^2 - 2(x + 2) = x^2 + 2x$

29. $f(y) = 3 - \sqrt{y}$

(a) $f(4) = 3 - \sqrt{4} = 1$

(b) $f(0.25) = 3 - \sqrt{0.25} = 2.5$

(c) $f(4x^2) = 3 - \sqrt{4x^2} = 3 - 2|x|$

30. $f(x) = \sqrt{x + 8} + 2$

(a) $f(-8) = \sqrt{(-8) + 8} + 2 = 2$

(b) $f(1) = \sqrt{(1) + 8} + 2 = 5$

(c) $f(x - 8) = \sqrt{(x - 8) + 8} + 2 = \sqrt{x} + 2$

31. $q(x) = \dfrac{1}{x^2 - 9}$

(a) $q(0) = \dfrac{1}{0^2 - 9} = -\dfrac{1}{9}$

(b) $q(3) = \dfrac{1}{3^2 - 9}$ is undefined.

(c) $q(y + 3) = \dfrac{1}{(y + 3)^2 - 9} = \dfrac{1}{y^2 + 6y}$

32. $q(t) = \dfrac{2t^2 + 3}{t^2}$

(a) $q(2) = \dfrac{2(2)^2 + 3}{(2)^2} = \dfrac{8 + 3}{4} = \dfrac{11}{4}$

(b) $q(0) = \dfrac{2(0)^2 + 3}{(0)^2}$

Division by zero is undefined.

(c) $q(-x) = \dfrac{2(-x)^2 + 3}{(-x)^2} = \dfrac{2x^2 + 3}{x^2}$

33. $f(x) = \dfrac{|x|}{x}$

(a) $f(2) = \dfrac{|2|}{2} = 1$

(b) $f(-2) = \dfrac{|-2|}{-2} = -1$

(c) $f(x-1) = \dfrac{|x-1|}{x-1} = \begin{cases} -1 & \text{if } x < 1 \\ 1 & \text{if } x > 1 \end{cases}$

34. $f(x) = |x| + 4$

(a) $f(2) = |2| + 4 = 6$

(b) $f(-2) = |-2| + 4 = 6$

(c) $f(x^2) = |x^2| + 4 = x^2 + 4$

35. $f(x) = \begin{cases} 2x+1, & x < 0 \\ 2x+2, & x \ge 0 \end{cases}$

(a) $f(-1) = 2(-1) + 1 = -1$

(b) $f(0) = 2(0) + 2 = 2$

(c) $f(2) = 2(2) + 2 = 6$

36. $f(x) = \begin{cases} x^2+2, & x \le 1 \\ 2x^2+2, & x > 1 \end{cases}$

(a) $f(-2) = (-2)^2 + 2 = 6$

(b) $f(1) = (1)^2 + 2 = 3$

(c) $f(2) = 2(2)^2 + 2 = 10$

37. $f(x) = \begin{cases} 3x-1, & x < -1 \\ 4, & -1 \le x \le 1 \\ x^2, & x > 1 \end{cases}$

(a) $f(-2) = 3(-2) - 1 = -7$

(b) $f\left(-\frac{1}{2}\right) = 4$

(c) $f(3) = 3^2 = 9$

38. $f(x) = \begin{cases} 4-5x, & x \le -2 \\ 0, & -2 < x \le 2 \\ x^2+1 & x > 2 \end{cases}$

(a) $f(-3) = 4 - 5(-3) = 19$

(b) $f(4) = (4)^2 + 1 = 17$

(c) $f(-1) = 0$

39. $f(x) = x^2 - 3$

$f(-2) = (-2)^2 - 3 = 1$

$f(-1) = (-1)^2 - 3 = -2$

$f(0) = (0)^2 - 3 = -3$

$f(1) = (1)^2 - 3 = -2$

$f(2) = (2)^2 - 3 = 1$

x	-2	-1	0	1	2
$f(x)$	1	-2	-3	-2	1

40. $g(x) = \sqrt{x-3}$

$g(3) = \sqrt{3-3} = 0$

$g(4) = \sqrt{4-3} = 1$

$g(5) = \sqrt{5-3} = \sqrt{2}$

$g(6) = \sqrt{6-3} = \sqrt{3}$

$g(7) = \sqrt{7-3} = 2$

x	3	4	5	6	7
$g(x)$	0	1	$\sqrt{2}$	$\sqrt{3}$	2

41. $h(t) = \dfrac{1}{2}|t+3|$

$h(-5) = \dfrac{1}{2}|-5+3| = 1$

$h(-4) = \dfrac{1}{2}|-4+3| = \dfrac{1}{2}$

$h(-3) = \dfrac{1}{2}|-3+3| = 0$

$h(-2) = \dfrac{1}{2}|-2+3| = \dfrac{1}{2}$

$h(-1) = \dfrac{1}{2}|-1+3| = 1$

t	-5	-4	-3	-2	-1
$h(t)$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	1

42. $f(s) = \dfrac{|s-2|}{s-2}$

$f(0) = \dfrac{|0-2|}{0-2} = \dfrac{2}{-2} = -1$

$f(1) = \dfrac{|1-2|}{1-2} = \dfrac{1}{-1} = -1$

$f\left(\dfrac{3}{2}\right) = \dfrac{|(3/2)-2|}{(3/2)-2} = \dfrac{1/2}{-1/2} = -1$

$f\left(\dfrac{5}{2}\right) = \dfrac{|(5/2)-2|}{(5/2)-2} = \dfrac{1/2}{1/2} = 1$

$f(4) = \dfrac{|4-2|}{4-2} = \dfrac{2}{2} = 1$

s	0	1	$\frac{3}{2}$	$\frac{5}{2}$	4
$f(s)$	-1	-1	-1	1	1

43. $f(x) = \begin{cases} -\frac{1}{2}x + 4, & x \le 0 \\ (x-2)^2, & x > 0 \end{cases}$

x	-2	-1	0	1	2
$f(x)$	5	$\frac{9}{2}$	4	1	0

$f(-2) = -\frac{1}{2}(-2) + 4 = 5$

$f(-1) = -\frac{1}{2}(-1) + 4 = 4\frac{1}{2} = \frac{9}{2}$

$f(0) = -\frac{1}{2}(0) + 4 = 4$

$f(1) = (1-2)^2 = 1$

$f(2) = (2-2)^2 = 0$

44. $f(x) = \begin{cases} 9 - x^2, & x < 3 \\ x - 3, & x \geq 3 \end{cases}$

$f(1) = 9 - (1)^2 = 8$

$f(2) = 9 - (2)^2 = 5$

$f(3) = (3) - 3 = 0$

$f(4) = (4) - 3 = 1$

$f(5) = (5) - 3 = 2$

x	1	2	3	4	5
$f(x)$	8	5	0	1	2

45. $15 - 3x = 0$

$3x = 15$

$x = 5$

46. $f(x) = 5x + 1$

$5x + 1 = 0$

$x = -\frac{1}{5}$

47. $\frac{3x - 4}{5} = 0$

$3x - 4 = 0$

$x = \frac{4}{3}$

48. $f(x) = \frac{12 - x^2}{5}$

$\frac{12 - x^2}{5} = 0$

$x^2 = 12$

$x = \pm\sqrt{12} = \pm 2\sqrt{3}$

49. $x^2 - 9 = 0$

$x^2 = 9$

$x = \pm 3$

50. $f(x) = x^2 - 8x + 15$

$x^2 - 8x + 15 = 0$

$(x - 5)(x - 3) = 0$

$x - 5 = 0 \implies x = 5$

$x - 3 = 0 \implies x = 3$

51. $x^3 - x = 0$

$x(x^2 - 1) = 0$

$x(x + 1)(x - 1) = 0$

$x = 0,\ x = -1$, or $x = 1$

52. $f(x) = x^3 - x^2 - 4x + 4$

$x^3 - x^2 - 4x + 4 = 0$

$x^2(x - 1) - 4(x - 1) = 0$

$(x - 1)(x^2 - 4) = 0$

$x - 1 = 0 \implies x = 1$

$x^2 - 4 = 0 \implies x = \pm 2$

53. $f(x) = g(x)$

$x^2 + 2x + 1 = 3x + 3$

$x^2 - x - 2 = 0$

$(x + 1)(x - 2) = 0$

$x = -1$ or $x = 2$

54. $f(x) = g(x)$

$x^4 - 2x^2 = 2x^2$

$x^4 - 4x^2 = 0$

$x^2(x^2 - 4) = 0$

$x^2(x + 2)(x - 2) = 0$

$x^2 = 0 \implies x = 0$

$x + 2 = 0 \implies x = -2$

$x - 2 = 0 \implies x = 2$

55. $f(x) = g(x)$

$\sqrt{3x} + 1 = x + 1$

$\sqrt{3x} = x$

$3x = x^2$

$0 = x^2 - 3x$

$0 = x(x - 3)$

$x = 0$ or $x = 3$

56. $f(x) = g(x)$

$\sqrt{x} - 4 = 2 - x$

$x + \sqrt{x} - 6 = 0$

$(\sqrt{x} + 3)(\sqrt{x} - 2) = 0$

$\sqrt{x} + 3 = 0 \implies \sqrt{x} = -3$, which is a contradiction, since $\sqrt{x}$ represents the principal square root.

$\sqrt{x} - 2 = 0 \implies \sqrt{x} = 2 \implies x = 4$

57. $f(x) = 5x^2 + 2x - 1$

Since $f(x)$ is a polynomial, the domain is all real numbers x.

58. $f(x) = 1 - 2x^2$

Because $f(x)$ is a polynomial, the domain is all real numbers x.

59. $h(t) = \dfrac{4}{t}$

Domain: All real numbers t except $t = 0$

60. $s(y) = \dfrac{3y}{y + 5}$

$y + 5 \neq 0$

$y \neq -5$

The domain is all real numbers y except $y = -5$.

61. $g(y) = \sqrt{y - 10}$

Domain: $y - 10 \geq 0$

$y \geq 10$

62. $f(t) = \sqrt[3]{t + 4}$

Because $f(t)$ is a cube root, the domain is all real numbers t.

63. $f(x) = \sqrt[4]{1 - x^2}$

Domain: $1 - x^2 \geq 0$

By solving this inequality, we conclude that $-1 \leq x \leq 1$ or $[-1, 1]$.

64. $f(x) = \sqrt[4]{x^2 + 3x}$

$x^2 + 3x \geq 0$

$x(x + 3) \geq 0$

By solving this inequality, we conclude that $x \leq -3$ or $x \geq 0$ or $(-\infty, -3] \cup [0, \infty)$.

65. $g(x) = \dfrac{1}{x} - \dfrac{3}{x + 2}$

Domain: All real numbers x except $x = 0,\ x = -2$

66. $h(x) = \dfrac{10}{x^2 - 2x}$

$x^2 - 2x \neq 0$

$x(x - 2) \neq 0$

The domain is all real numbers x except $x = 0, x = 2$

67. $f(s) = \dfrac{\sqrt{s - 1}}{s - 4}$

Domain: $s - 1 \geq 0 \implies s \geq 1$ and $s \neq 4$

The domain consists of all real numbers s, such that $s \geq 1$ and $s \neq 4$.

68. $f(x) = \dfrac{\sqrt{x + 6}}{6 + x}$

Domain: $x + 6 \geq 0 \implies x \geq -6$ and $x \neq -6$

The domain is all real numbers x such that $x > -6$ or $(-6, \infty)$.

69. $f(x) = \dfrac{x - 4}{\sqrt{x}}$

The domain is all real numbers such that $x > 0$ or $(0, \infty)$.

70. $f(x) = \dfrac{x - 5}{\sqrt{x^2 - 9}}$

$x^2 - 9 > 0$

$(x + 3)(x - 3) > 0$

Test intervals:
$(-\infty, -3), (-3, 3), (3, \infty)$

The domain is all real numbers $x < -3$ or $x > 3$ or $(-\infty, -3) \cup (3, \infty)$.

71. $f(x) = x^2$

$\{(-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)\}$

72. $f(x) = x^2 - 3$

$f(-2) = (-2)^2 - 3 = 1$

$f(-1) = (-1)^2 - 3 = -2$

$f(0) = (0)^2 - 3 = -3$

$f(1) = (1)^2 - 3 = -2$

$f(2) = (2)^2 - 3 = 1$

$\{(-2, 1), (-1, -2), (0, -3), (1, -2), (2, 1)\}$

73. $f(x) = |x| + 2$

$\{(-2, 4), (-1, 3), (0, 2), (1, 3), (2, 4)\}$

74. $f(x) = |x + 1|$

$\{(-2, 1), (-1, 0), (0, 1), (1, 2), (2, 3)\}$

75. By plotting the points, we have a parabola, so $g(x) = cx^2$. Since $(-4, -32)$ is on the graph, we have $-32 = c(-4)^2 \implies c = -2$. Thus, $g(x) = -2x^2$.

76. By plotting the data, you can see that they represent a line, or $f(x) = cx$. Because $(0, 0)$ and $\left(1, \frac{1}{4}\right)$ are on the line, the slope is $\frac{1}{4}$. Thus, $f(x) = \frac{1}{4}x$.

77. Since the function is undefined at 0, we have $r(x) = c/x$. Since $(-4, -8)$ is on the graph, we have $-8 = c/-4 \implies c = 32$. Thus, $r(x) = 32/x$.

78. By plotting the data, you can see that they represent $h(x) = c\sqrt{|x|}$. Because $\sqrt{|-4|} = 2$ and $\sqrt{|-1|} = 1$, and the corresponding y-values are 6 and 3, $c = 3$ and $h(x) = 3\sqrt{|x|}$.

79.

$$\begin{aligned}
f(x) &= x^2 - x + 1\\
f(2 + h) &= (2 + h)^2 - (2 + h) + 1\\
&= 4 + 4h + h^2 - 2 - h + 1\\
&= h^2 + 3h + 3\\
f(2) &= (2)^2 - 2 + 1 = 3\\
f(2 + h) - f(2) &= h^2 + 3h\\
\frac{f(2 + h) - f(2)}{h} &= \frac{h^2 + 3h}{h} = h + 3,\ h \neq 0
\end{aligned}$$

80.

$$\begin{aligned}
f(x) &= 5x - x^2\\
f(5 + h) &= 5(5 + h) - (5 + h)^2\\
&= 25 + 5h - (25 + 10h + h^2)\\
&= 25 + 5h - 25 - 10h - h^2\\
&= -h^2 - 5h\\
f(5) &= 5(5) - (5)^2\\
&= 25 - 25 = 0\\
\frac{f(5 + h) - f(5)}{h} &= \frac{-h^2 - 5h}{h}\\
&= \frac{-h(h + 5)}{h} = -(h + 5), h \neq 0
\end{aligned}$$

81.

$$\begin{aligned}
f(x) &= x^3 + 3x\\
f(x + h) &= (x + h)^3 + 3(x + h)\\
&= x^3 + 3x^2h + 3xh^2 + h^3 + 3x + 3h\\
\frac{f(x + h) - f(x)}{h} &= \frac{(x^3 + 3x^2h + 3xh^2 + h^3 + 3x + 3h) - (x^3 + 3x)}{h}\\
&= \frac{h(3x^2 + 3xh + h^2 + 3)}{h}\\
&= 3x^2 + 3xh + h^2 + 3,\quad h \neq 0
\end{aligned}$$

82.

$$\begin{aligned}
f(x) &= 4x^2 - 2x\\
f(x + h) &= 4(x + h)^2 - 2(x + h)\\
&= 4(x^2 + 2xh + h^2) - 2x - 2h\\
&= 4x^2 + 8xh + 4h^2 - 2x - 2h\\
\frac{f(x + h) - f(x)}{h} &= \frac{4x^2 + 8xh + 4h^2 - 2x - 2h - 4x^2 + 2x}{h}\\
&= \frac{8xh + 4h^2 - 2h}{h}\\
&= \frac{h(8x + 4h - 2)}{h}\\
&= 8x + 4h - 2,\quad h \neq 0
\end{aligned}$$

83. $g(x) = \dfrac{1}{x^2}$

$$\begin{aligned}
\frac{g(x) - g(3)}{x - 3} &= \frac{\frac{1}{x^2} - \frac{1}{9}}{x - 3}\\
&= \frac{9 - x^2}{9x^2(x - 3)}\\
&= \frac{-(x + 3)(x - 3)}{9x^2(x - 3)}\\
&= -\frac{x + 3}{9x^2},\ x \neq 3
\end{aligned}$$

84.
$$f(t) = \frac{1}{t-2}$$
$$f(1) = \frac{1}{1-2} = -1$$
$$\frac{f(t) - f(1)}{t-1} = \frac{\frac{1}{t-2} - (-1)}{t-1} = \frac{1 + (t-2)}{(t-2)(t-1)} = \frac{(t-1)}{(t-2)(t-1)} = \frac{1}{t-2}, \quad t \neq 1$$

85. $f(x) = \sqrt{5x}$
$$\frac{f(x) - f(5)}{x-5} = \frac{\sqrt{5x} - 5}{x-5}$$

86.
$$f(x) = x^{2/3} + 1$$
$$f(8) = 8^{2/3} + 1 = 5$$
$$\frac{f(x) - f(8)}{x-8} = \frac{x^{2/3} + 1 - 5}{x-8} = \frac{x^{2/3} - 4}{x-8}$$

87. $A = s^2$ and $P = 4s \implies \frac{P}{4} = s$
$$A = \left(\frac{P}{4}\right)^2 = \frac{P^2}{16}$$

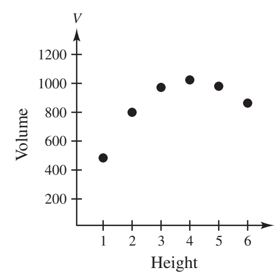
88. $A = \pi r^2, \ C = 2\pi r$
$$r = \frac{C}{2\pi}$$
$$A = \pi\left(\frac{C}{2\pi}\right)^2 = \frac{C^2}{4\pi}$$

89. (a)

Height, x	Volume, V
1	484
2	800
3	972
4	1024
5	980
6	864

The volume is maximum when $x = 4$ and $V = 1024$ cubic centimeters.

(b)



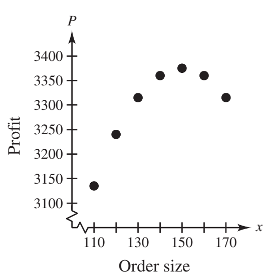
V is a function of x.

(c) $V = x(24 - 2x)^2$

Domain: $0 < x < 12$

90. (a) The maximum profit is \$3375.

(b)



Yes, P is a function of x.

(c) Profit = Revenue − Cost

$$\begin{aligned}
&= (\text{price per unit})(\text{number of units}) - (\text{cost})(\text{number of units}) \\
&= [90 - (x - 100)(0.15)]x - 60x, \ x > 100 \\
&= (90 - 0.15x + 15)x - 60x \\
&= (105 - 0.15x)x - 60x \\
&= 105x - 0.15x^2 - 60x \\
&= 45x - 0.15x^2, \ x > 100
\end{aligned}$$

91. $A = \frac{1}{2}bh = \frac{1}{2}xy$

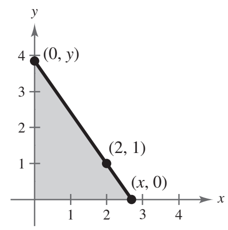
Since $(0, y)$, $(2, 1)$, and $(x, 0)$ all lie on the same line, the slopes between any pair are equal.

$$\frac{1-y}{2-0} = \frac{0-1}{x-2}$$

$$\frac{1-y}{2} = \frac{-1}{x-2}$$

$$y = \frac{2}{x-2} + 1$$

$$y = \frac{x}{x-2}$$



Therefore,

$$A = \frac{1}{2}x\left(\frac{x}{x-2}\right) = \frac{x^2}{2(x-2)}.$$

The domain of A includes x-values such that $x^2/[2(x-2)] > 0$. By solving this inequality, we find that the domain is $x > 2$.

92. $A = l \cdot w = (2x)y = 2xy$

But $y = \sqrt{36 - x^2}$, so $A = 2x\sqrt{36 - x^2}$, $0 < x < 6$.

93. $y = -\frac{1}{10}x^2 + 3x + 6$

$y(30) = -\frac{1}{10}(30)^2 + 3(30) + 6 = 6$ feet

If the child holds a glove at a height of 5 feet, then the ball *will* be over the child's head since it will be at a height of 6 feet.

94. $d(t) = \begin{cases} 5.0t + 37, & 0 \le t \le 7 \\ 18.7t - 64, & 0 \le t \le 12 \end{cases}$ where $t = 1$ represents 1991.

1991: $t = 1$ and $d(1) = 5.0(1) + 37 = 42$ billion dollars $= \$42{,}000{,}000{,}000$

1992: $t = 2$ and $d(2) = 5.0(2) + 37 = 47$ billion dollars $= \$47{,}000{,}000{,}000$

1993: $t = 3$ and $d(3) = 5.0(3) + 37 = 52$ billion dollars $= \$52{,}000{,}000{,}000$

1994: $t = 4$ and $d(4) = 5.0(4) + 37 = 57$ billion dollars $= \$57{,}000{,}000{,}000$

1995: $t = 5$ and $d(5) = 5.0(5) + 37 = 62$ billion dollars $= \$62{,}000{,}000{,}000$

1996: $t = 6$ and $d(6) = 5.0(6) + 37 = 67$ billion dollars $= \$67{,}000{,}000{,}000$

1997: $t = 7$ and $d(7) = 5.0(7) + 37 = 72$ billion dollars $= \$72{,}000{,}000{,}000$

1998: $t = 8$ and $d(8) = 18.7(8) - 64 = 85.6$ billion dollars $= \$85{,}600{,}000{,}000$

1999: $t = 9$ and $d(9) = 18.7(9) - 64 = 104.3$ billion dollars $= \$104{,}300{,}000{,}000$

2000: $t = 10$ and $d(10) = 18.7(10) - 64 = 123$ billion dollars $= \$123{,}000{,}000{,}000$

2001: $t = 11$ and $d(11) = 18.7(11) - 64 = 141.7$ billion dollars $= \$141{,}700{,}000{,}000$

2002: $t = 12$ and $d(12) = 18.7(12) - 64 = 160.4$ billion dollars $= \$160{,}400{,}000{,}000$

95. $p(t) = \begin{cases} 0.182t^2 + 0.57t + 27.3, & 0 \le t \le 7 \\ 2.50t + 21.3, & 8 \le t \le 12 \end{cases}$

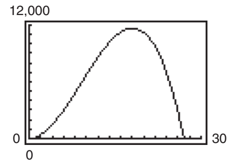
Year	Function Value	Price
1990	$p(0) = 27.3$	\$27,300
1991	$p(1) = 28.052$	\$28,052
1992	$p(2) = 29.168$	\$29,168
1993	$p(3) = 30.648$	\$30,648
1994	$p(4) = 32.492$	\$32,492
1995	$p(5) = 34.7$	\$34,700
1996	$p(6) = 37.272$	\$37,272
1997	$p(7) = 40.208$	\$40,208
1998	$p(8) = 41.3$	\$41,300
1999	$p(9) = 43.8$	\$43,800
2000	$p(10) = 46.3$	\$46,300
2001	$p(11) = 48.8$	\$48,800
2002	$p(12) = 51.3$	\$51,300

96. (a) $V = l \cdot w \cdot h = x \cdot y \cdot x = x^2y$ where $4x + y = 108$. Thus, $y = 108 - 4x$ and

$V = x^2(108 - 4x) = 108x^2 - 4x^3$.

Domain: $0 < x < 27$

(b)



(c) The dimensions that will maximize the volume of the package are $18 \times 18 \times 36$. From the graph, the maximum volume occurs when $x = 18$. To find the dimension for y, use the equation $y = 108 - 4x$.

$y = 108 - 4x = 108 - 4(18) = 108 - 72 = 36$

97. (a) Cost = variable costs + fixed costs

$C = 12.30x + 98{,}000$

(b) Revenue = price per unit × number of units

$R = 17.98x$

(c) Profit = Revenue − Cost

$P = 17.98x - (12.30x + 98{,}000)$

$P = 5.68x - 98{,}000$

98. (a) *Model:* (Total cost) = (Fixed costs) + (Variable costs)

Labels: Total cost $= C$

Fixed cost $= 6000$

Variable costs $= 0.95x$

Equation: $C = 6000 + 0.95x$

(b) $\overline{C} = \dfrac{C}{x} = \dfrac{6000 + 0.95x}{x}$

99. (a) $R = n(\text{rate}) = n[8.00 - 0.05(n - 80)],\ n \ge 80$

$$R = 12.00n - 0.05n^2 = 12n - \frac{n^2}{20} = \frac{240n - n^2}{20},\ n \ge 80$$

(b)

n	90	100	110	120	130	140	150
$R(n)$	\$675	\$700	\$715	\$720	\$715	\$700	\$675

The revenue is maximum when 120 people take the trip.

100. $F(y) = 149.76\sqrt{10}y^{5/2}$

(a)

y	5	10	20	30	40
$F(y)$	26,474.08	149,760.00	847,170.49	2,334,527.36	4,792,320

The force, in tons, of the water against the dam increases with the depth of the water.

(b) It appears that approximately 21 feet of water would produce 1,000,000 tons of force.

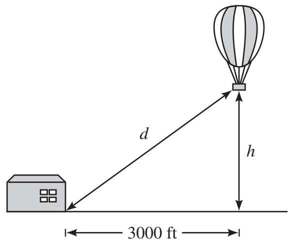
(c) $1{,}000{,}000 = 149.76\sqrt{10}y^{5/2}$

$$\frac{1{,}000{,}000}{149.76\sqrt{10}} = y^{5/2}$$

$2111.56 \approx y^{5/2}$

$21.37 \text{ feet} \approx y$

101. (a)



(b) $(3000)^2 + h^2 = d^2$

$$h = \sqrt{d^2 - (3000)^2}$$

Domain: $d \geq 3000$ (since both $d \geq 0$ and $d^2 - (3000)^2 \geq 0$)

102. (a) $\dfrac{f(2003) - f(1996)}{2003 - 1996} = \dfrac{126 - 116}{7} = \dfrac{10}{7} = 1.428$

The number of threatened and endangered fish species increased, on average, by 1.428 per year from 1996 to 2003.

(b) $y = \begin{cases} 2x + 104, & 6 \leq x \leq 7 \\ 2x + 103, & 8 \leq x \leq 11 \\ 126, & 12 \leq x \leq 13 \end{cases}$

(c)

Year $6 \leftrightarrow 1996$	Actual Number of Fish Species	Number from the Algebraic Model	Number from the Calculator Model
6	116	116	116
7	118	118	118
8	119	119	119
9	121	121	121
10	123	123	122
11	125	125	124
12	126	126	126
13	126	126	127

(d) The algebraic model is an excellent fit to the actual data.

(e) The calculator model is

$y \approx 1.55x + 107.$

It also gives a good fit, but not as good as the algebraic model.

103. False. The range is $[-1, \infty)$.

104. True. The set represents a function. Each x-value is mapped to exactly one y-value.

105. The domain is the set of inputs of the function, and the range is the set of outputs.

106. Since $f(x)$ is a function of an even root, the radicand cannot be negative. $g(x)$ is an odd root, therefore the radicand can be any real number. Therefore, the domain of f is all real numbers x and the domain of g is all real numbers x such that $x \geq 2$.

107. (a) Yes. The amount that you pay in sales tax will increase as the price of the item purchased increases.

(b) No. The length of time that you study the night before an exam does not necessarily determine your score on the exam.

108. (a) No. During the course of a year, for example, your salary may remain constant while your savings account balance may vary. That is, there may be two or more outputs (savings account balances) for one input (salary).

(b) Yes. The greater the height from which the ball is dropped, the greater the speed with which the ball will strike the ground.

109.

$$\frac{t}{3} + \frac{t}{5} = 1$$

$$15\left(\frac{t}{3} + \frac{t}{5}\right) = 15(1)$$

$$5t + 3t = 15$$

$$8t = 15$$

$$t = \frac{15}{8}$$

110. $\dfrac{3}{t} + \dfrac{5}{t} = 1$

$$\frac{8}{t} = 1$$

$$8 = t$$

111. $$\frac{3}{x(x+1)} - \frac{4}{x} = \frac{1}{x+1}$$

$$x(x+1)\left[\frac{3}{x(x+1)} - \frac{4}{x}\right] = x(x+1)\left(\frac{1}{x+1}\right)$$

$$3 - 4(x+1) = x$$

$$3 - 4x - 4 = x$$

$$-1 = 5x$$

$$-\frac{1}{5} = x$$

112. $$\frac{12}{x} - 3 = \frac{4}{x} + 9$$

$$\frac{12}{x} - \frac{4}{x} = 9 + 3$$

$$\frac{8}{x} = 12$$

$$\frac{8}{12} = x$$

$$x = \frac{2}{3}$$

113. $(-2, -5)$ and $(4, -1)$

$$m = \frac{-1-(-5)}{4-(-2)} = \frac{4}{6} = \frac{2}{3}$$

$$y - (-5) = \frac{2}{3}(x - (-2))$$

$$y + 5 = \frac{2}{3}x + \frac{4}{3}$$

$$3y + 15 = 2x + 4$$

$$2x - 3y - 11 = 0$$

114. $$\text{Slope} = \frac{9-0}{1-10} = \frac{9}{-9} = -1$$

$$m = -1$$

$$y - 0 = (-1)(x - 10)$$

$$y = -x + 10$$

$$x + y - 10 = 0$$

115. $(-6, 5)$ and $(3, -5)$

$$m = \frac{-5-5}{3-(-6)} = -\frac{10}{9}$$

$$y - 5 = -\frac{10}{9}(x - (-6))$$

$$9y - 45 = -10x - 60$$

$$10x + 9y + 15 = 0$$

116. $$\text{Slope} = \frac{-(1/3) - 3}{(11/2) - (-1/2)}$$

$$= \frac{-10/3}{12/2} = -\frac{10}{3} \cdot \frac{1}{6} = -\frac{5}{9}$$

$$m = -\frac{5}{9}$$

$$y - 3 = -\frac{5}{9}\left(x - \left(-\frac{1}{2}\right)\right)$$

$$y - 3 = -\frac{5}{9}x - \frac{5}{18}$$

$$18y - 54 = -10x - 5$$

$$10x + 18y - 49 = 0$$

Section 1.5 Analyzing Graphs of Functions

- You should be able to determine the domain and range of a function from its graph.
- You should be able to use the vertical line test for functions.
- You should be able to find the zeros of a function.
- You should be able to determine when a function is constant, increasing, or decreasing.
- You should be able to approximate relative minimums and relative maximums from the graph of a function.
- You should know that f is

  (a) odd if $f(-x) = -f(x)$. (b) even if $f(-x) = f(x)$.

Vocabulary Check

1. ordered pairs
2. vertical line test
3. zeros
4. decreasing
5. maximum
6. average rate of change; secant
7. odd
8. even

1. Domain: $(-\infty, -1] \cup [1, \infty)$

Range: $[0, \infty)$

2. Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

3. Domain: $[-4, 4]$

Range: $[0, 4]$

4. Domain: $(-\infty, 1), (1, \infty)$

Range: $-1, 1$

5. (a) $f(-2) = 0$ (b) $f(-1) = -1$

(c) $f\left(\frac{1}{2}\right) = 0$ (d) $f(1) = -3$

6. (a) $f(-1) = 4$ (b) $f(2) = 4$

(c) $f(0) = 2$ (d) $f(1) = 0$

7. (a) $f(-2) = -3$ (b) $f(1) = 0$

(c) $f(0) = 1$ (d) $f(2) = -3$

8. (a) $f(2) = 0$ (b) $f(1) = 1$

(c) $f(3) = 2$ (d) $f(-1) = 3$

9. $y = \frac{1}{2}x^2$

A vertical line intersects the graph just once, so y *is* a function of x.

10. $y = \frac{1}{4}x^3$

A vertical line intersects the graph no more than once, so y *is* a function of x.

11. $x - y^2 = 1 \implies y = \pm\sqrt{x - 1}$

y *is not* a function of x. Some vertical lines cross the graph twice.

12. $x^2 + y^2 = 25$

A vertical line intersects the graph more than once, so y *is not* a function of x.

13. $x^2 = 2xy - 1$

A vertical line intersects the graph just once, so y *is* a function of x.

14. $x = |y + 2|$

A vertical line intersects the graph more than once, so y *is not* a function of x.

15.
$$\begin{aligned} 2x^2 - 7x - 30 &= 0 \\ (2x + 5)(x - 6) &= 0 \\ 2x + 5 = 0 \quad &\text{or} \quad x - 6 = 0 \\ x = -\frac{5}{2} \quad &\text{or} \quad x = 6 \end{aligned}$$

16.
$$\begin{aligned} f(x) &= 3x^2 + 22x - 16 \\ 0 &= (3x - 2)(x + 8) \\ 3x - 2 = 0 &\implies x = \frac{2}{3} \\ x + 8 = 0 &\implies x = -8 \end{aligned}$$

17.
$$\frac{x}{9x^2 - 4} = 0$$
$$x = 0$$

18.
$$\begin{aligned} f(x) &= \frac{x^2 - 9x + 14}{4x} \\ 0 &= \frac{x^2 - 9x + 14}{4x} \\ 0 &= (x - 7)(x - 2) \\ x - 7 = 0 &\implies x = 7 \\ x - 2 = 0 &\implies x = 2 \end{aligned}$$

19.
$$\begin{aligned} \frac{1}{2}x^3 - x &= 0 \\ x^3 - 2x &= 2(0) \\ x(x^2 - 2) &= 0 \\ x = 0 \quad \text{or} \quad x^2 - 2 &= 0 \\ x^2 &= 2 \\ x &= \pm\sqrt{2} \end{aligned}$$

20.
$$\begin{aligned} f(x) &= x^3 - 4x^2 - 9x + 36 \\ 0 &= x^3 - 4x^2 - 9x + 36 \\ 0 &= x^2(x - 4) - 9(x - 4) \\ 0 &= (x - 4)(x^2 - 9) \\ x - 4 = 0 &\implies x = 4 \\ x^2 - 9 = 0 &\implies x = \pm 3 \end{aligned}$$

21.
$$\begin{aligned} 4x^3 - 24x^2 - x + 6 &= 0 \\ 4x^2(x - 6) - 1(x - 6) &= 0 \\ (x - 6)(4x^2 - 1) &= 0 \\ (x - 6)(2x + 1)(2x - 1) &= 0 \end{aligned}$$
$$x - 6 = 0, \quad 2x + 1 = 0, \quad 2x - 1 = 0$$
$$x = 6, \quad x = -\tfrac{1}{2}, \quad x = \tfrac{1}{2}$$

22.
$$\begin{aligned} f(x) &= 9x^4 - 25x^2 \\ 0 &= 9x^4 - 25x^2 \\ 0 &= x^2(9x^2 - 25) \\ x^2 = 0 &\implies x = 0 \\ 9x^2 - 25 = 0 &\implies x = \pm\tfrac{5}{3} \end{aligned}$$

23. $\sqrt{2x} - 1 = 0$

$\sqrt{2x} = 1$

$2x = 1$

$x = \frac{1}{2}$

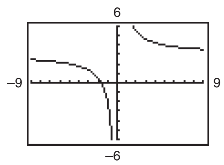
24. $f(x) = \sqrt{3x + 2}$

$0 = \sqrt{3x + 2}$

$0 = 3x + 2$

$-\frac{2}{3} = x$

25. (a)



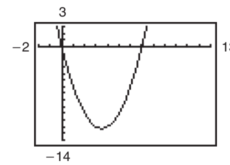
Zero: $x = -\dfrac{5}{3}$

(b) $3 + \dfrac{5}{x} = 0$

$3x + 5 = 0$

$x = -\dfrac{5}{3}$

26. (a)



Zeros: $x = 0, x = 7$

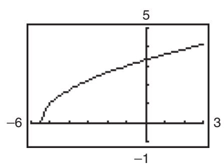
(b) $f(x) = x(x - 7)$

$0 = x(x - 7)$

$x = 0$

$x - 7 = 0 \Rightarrow x = 7$

27. (a)



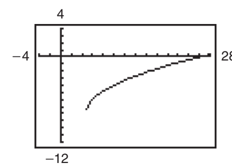
Zero: $x = -\frac{11}{2}$

(b) $\sqrt{2x + 11} = 0$

$2x + 11 = 0$

$x = -\frac{11}{2}$

28. (a)



Zero: $x = 26$

(b) $f(x) = \sqrt{3x - 14} - 8$

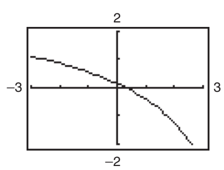
$0 = \sqrt{3x - 14} - 8$

$8 = \sqrt{3x - 14}$

$64 = 3x - 14$

$x = 26$

29. (a)



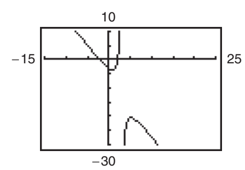
Zero: $x = \dfrac{1}{3}$

(b) $\dfrac{3x - 1}{x - 6} = 0$

$3x - 1 = 0$

$x = \dfrac{1}{3}$

30. (a)



Zeros: $x = \pm 2.1213$

(b) $f(x) = \dfrac{2x^2 - 9}{3 - x}$

$0 = \dfrac{2x^2 - 9}{3 - x}$

$2x^2 - 9 = 0 \Rightarrow x = \pm\dfrac{3\sqrt{2}}{2} = \pm 2.1213$

31. $f(x) = \frac{3}{2}x$

f is increasing on $(-\infty, \infty)$.

32. $f(x) = x^2 - 4x$

The graph is decreasing on $(-\infty, 2)$ and increasing on $(2, \infty)$.

33. $f(x) = x^3 - 3x^2 + 2$

f is increasing on $(-\infty, 0)$ and $(2, \infty)$.

f is decreasing on $(0, 2)$.

34. $f(x) = \sqrt{x^2 - 1}$

The graph is decreasing on $(-\infty, -1)$ and increasing on $(1, \infty)$.

35. $f(x) = \begin{cases} x + 3, & x \le 0 \\ 3, & 0 < x \le 2 \\ 2x + 1, & x > 2 \end{cases}$

f is increasing on $(-\infty, 0)$ and $(2, \infty)$.

f is constant on $(0, 2)$.

36. $f(x) = \begin{cases} 2x + 1, & x \le -1 \\ x^2 - 2, & x > -1 \end{cases}$

The graph is decreasing on $(-1, 0)$ and increasing on $(-\infty, -1)$ and $(0, \infty)$.

37. $f(x) = |x + 1| + |x - 1|$

f is increasing on $(1, \infty)$.

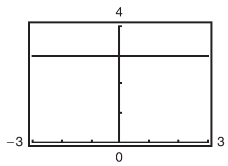
f is constant on $(-1, 1)$.

f is decreasing on $(-\infty, -1)$.

38. The graph is decreasing on $(-2, -1)$ and $(-1, 0)$ and increasing on $(-\infty, -2)$ and $(0, \infty)$.

39. $f(x) = 3$

(a)



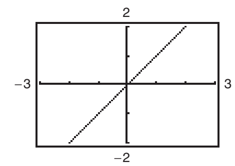
Constant on $(-\infty, \infty)$

(b)

x	-2	-1	0	1	2
$f(x)$	3	3	3	3	3

40. $g(x) = x$

(a)



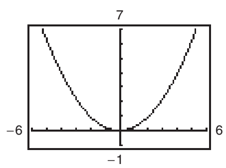
Increasing on $(-\infty, \infty)$

(b)

x	-2	-1	0	1	2
$g(x)$	-2	-1	0	1	2

41. $g(s) = \dfrac{s^2}{4}$

(a)



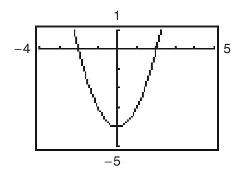
Decreasing on $(-\infty, 0)$; Increasing on $(0, \infty)$

(b)

s	-4	-2	0	2	4
$g(s)$	4	1	0	1	4

42. $h(x) = x^2 - 4$

(a)



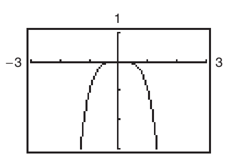
Decreasing on $(-\infty, 0)$; Increasing on $(0, \infty)$

(b)

x	-2	-1	0	1	2
$h(x)$	0	-3	-4	-3	0

43. $f(t) = -t^4$

(a)



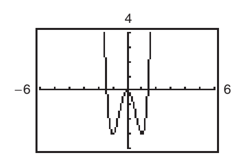
Increasing on $(-\infty, 0)$; Decreasing on $(0, \infty)$

(b)

t	-2	-1	0	1	2
$f(t)$	-16	-1	0	-1	-16

44. $f(x) = 3x^4 - 6x^2$

(a)



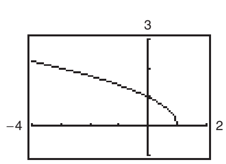
Increasing on $(-1, 0)$, $(1, \infty)$; Decreasing on $(-\infty, -1)$, $(0, 1)$

(b)

x	-2	-1	0	1	2
$f(x)$	24	-3	0	-3	24

45. $f(x) = \sqrt{1 - x}$

(a)



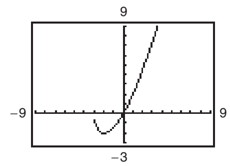
Decreasing on $(-\infty, 1)$

(b)

x	-3	-2	-1	0	1
$f(x)$	2	$\sqrt{3}$	$\sqrt{2}$	1	0

46. $f(x) = x\sqrt{x + 3}$

(a)



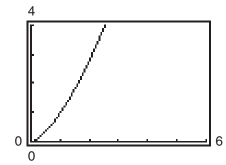
Increasing on $(-2, \infty)$; Decreasing on $(-3, -2)$

(b)

x	-3	-2	-1	0	1
$f(x)$	0	-2	-1.414	0	2

47. $f(x) = x^{3/2}$

(a)



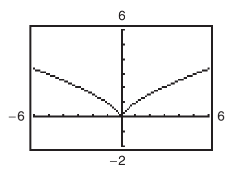
Increasing on $(0, \infty)$

(b)

x	0	1	2	3	4
$f(x)$	0	1	2.8	5.2	8

48. $f(x) = x^{2/3}$

(a)

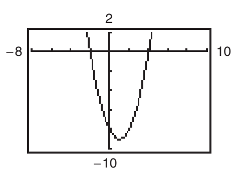


Decreasing on $(-\infty, 0)$; Increasing on $(0, \infty)$

(b)

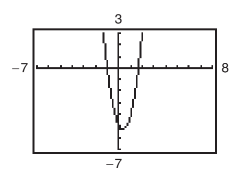
x	-2	-1	0	1	2
$f(x)$	1.59	1	0	1	1.59

49. $f(x) = (x - 4)(x + 2)$



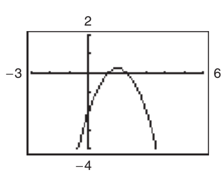
Relative minimum: $(1, -9)$

50. $f(x) = 3x^2 - 2x - 5$



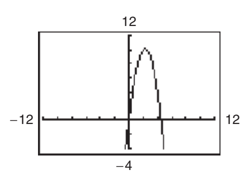
Relative minimum: $\left(\frac{1}{3}, -\frac{16}{3}\right)$ or $(0.33, -5.33)$

51. $f(x) = -x^2 + 3x - 2$



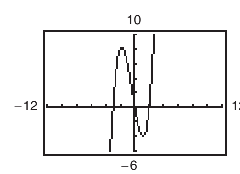
Relative maximum: $(1.5, 0.25)$

52. $f(x) = -2x^2 + 9x$



Relative maximum: $(2.25, 10.125)$

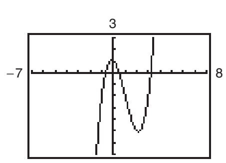
53. $f(x) = x(x - 2)(x + 3)$



Relative minimum: $(1.12, -4.06)$

Relative maximum: $(-1.79, 8.21)$

54. $f(x) = x^3 - 3x^2 - x + 1$

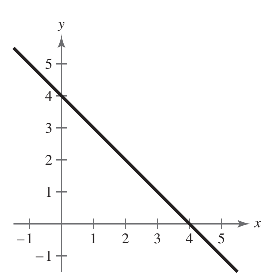


Relative maximum: $(-0.15, 1.08)$

Relative minimum: $(2.15, -5.08)$

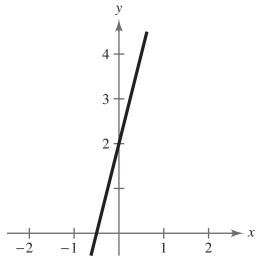
55. $f(x) = 4 - x$

$f(x) \geq 0$ on $(-\infty, 4]$.



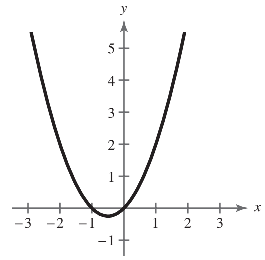
56. $f(x) = 4x + 2$

$$f(x) \geq 0$$
$$4x + 2 \geq 0$$
$$4x \geq -2$$
$$x \geq -\tfrac{1}{2}$$
$$\left[-\tfrac{1}{2}, \infty\right)$$



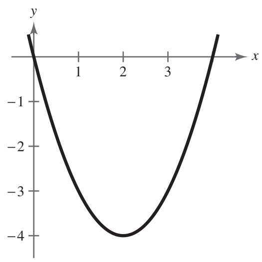
57. $f(x) = x^2 + x$

$f(x) \geq 0$ on $(-\infty, -1]$ and $[0, \infty)$.



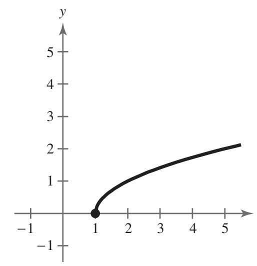
58. $f(x) = x^2 - 4x$

$$f(x) \geq 0$$
$$x^2 - 4x \geq 0$$
$$x(x - 4) \geq 0$$
$$(-\infty, 0], [4, \infty)$$



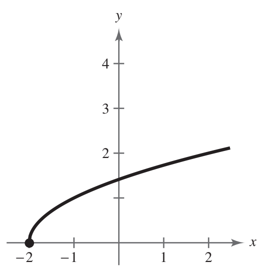
59. $f(x) = \sqrt{x - 1}$

$f(x) \geq 0$ on $[1, \infty)$.



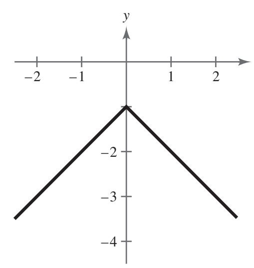
60. $f(x) = \sqrt{x + 2}$

$$f(x) \geq 0$$
$$\sqrt{x + 2} \geq 0$$
$$x + 2 \geq 0$$
$$x \geq -2$$
$$[-2, \infty)$$



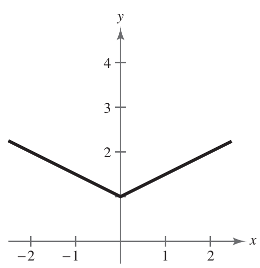
61. $f(x) = -(1 + |x|)$

$f(x)$ is never greater than 0. ($f(x) < 0$ for all x.)



62. $f(x) = \frac{1}{2}(2 + |x|)$

$f(x)$ is always greater than 0. $(-\infty, \infty)$



63. $f(x) = -2x + 15$

$$\frac{f(3) - f(0)}{3 - 0} = \frac{9 - 15}{3} = -2$$

The average rate of change from $x_1 = 0$ to $x_2 = 3$ is -2.

64. $f(x) = 3x + 8$

$$\frac{f(3) - f(0)}{3 - 0} = \frac{17 - 8}{3} = \frac{9}{3} = 3$$

The average rate of change from $x_1 = 0$ to $x_2 = 3$ is 3.

65. $f(x) = x^2 + 12x - 4$

$$\frac{f(5) - f(1)}{5 - 1} = \frac{81 - 9}{4} = 18$$

The average rate of change from $x_1 = 1$ to $x_2 = 5$ is 18.

66. $f(x) = x^2 - 2x + 8$

$$\frac{f(5) - f(1)}{5 - 1} = \frac{23 - 7}{4} = \frac{16}{4} = 4$$

The average rate of change from $x_1 = 1$ to $x_2 = 5$ is 4.

67. $f(x) = x^3 - 3x^2 - x$

$$\frac{f(3) - f(1)}{3 - 1} = \frac{-3 - (-3)}{2} = 0$$

The average rate of change from $x_1 = 1$ to $x_2 = 3$ is 0.

68. $f(x) = -x^3 + 6x^2 + x$

$$\frac{f(6) - f(1)}{6 - 1} = \frac{6 - 6}{5} = \frac{0}{5} = 0$$

The average rate of change from $x_1 = 1$ to $x_2 = 6$ is 0.

69. $f(x) = -\sqrt{x - 2} + 5$

$$\frac{f(11) - f(3)}{11 - 3} = \frac{2 - 4}{8} = -\frac{1}{4}$$

The average rate of change from $x_1 = 3$ to $x_2 = 11$ is $-\frac{1}{4}$.

70. $f(x) = -\sqrt{x + 1} + 3$

$$\frac{f(8) - f(3)}{8 - 3} = \frac{0 - 1}{5} = -\frac{1}{5}$$

The average rate of change from $x_1 = 3$ to $x_2 = 8$ is $-\frac{1}{5}$.

71. $f(x) = x^6 - 2x^2 + 3$

$$\begin{aligned} f(-x) &= (-x)^6 - 2(-x)^2 + 3 \\ &= x^6 - 2x^2 + 3 \\ &= f(x) \end{aligned}$$

The function is even.
y-axis symmetry

72. $h(x) = x^3 - 5$

$$\begin{aligned} h(-x) &= (-x)^3 - 5 \\ &= -x^3 - 5 \\ &\neq h(x) \\ &\neq -h(x) \end{aligned}$$

The function is neither odd nor even. No symmetry

73. $g(x) = x^3 - 5x$

$$\begin{aligned} g(-x) &= (-x)^3 - 5(-x) \\ &= -x^3 + 5x \\ &= -g(x) \end{aligned}$$

The function is odd.
Origin symmetry

74. $f(x) = x\sqrt{1 - x^2}$

$$\begin{aligned} f(-x) &= -x\sqrt{1 - (-x)^2} \\ &= -x\sqrt{1 - x^2} \\ &= -f(x) \end{aligned}$$

The function is odd.
Origin symmetry

75. $f(t) = t^2 + 2t - 3$

$$\begin{aligned} f(-t) &= (-t)^2 + 2(-t) - 3 \\ &= t^2 - 2t - 3 \\ &\neq f(t), \neq -f(t) \end{aligned}$$

The function is neither even nor odd. No symmetry

76. $g(s) = 4s^{2/3}$

$$\begin{aligned} g(-s) &= 4(-s)^{2/3} \\ &= 4s^{2/3} \\ &= g(s) \end{aligned}$$

The function is even.
y-axis symmetry

77. $h = \text{top} - \text{bottom}$

$$\begin{aligned} &= (-x^2 + 4x - 1) - 2 \\ &= -x^2 + 4x - 3 \end{aligned}$$

78. $h = \text{top} - \text{bottom}$

$$\begin{aligned} &= 3 - (4x - x^2) \\ &= 3 - 4x + x^2 \end{aligned}$$

79. $h = \text{top} - \text{bottom}$

$$\begin{aligned} &= (4x - x^2) - 2x \\ &= 2x - x^2 \end{aligned}$$

80. $h = \text{top} - \text{bottom}$

$$= 2 - \sqrt[3]{x}$$

81. $L = \text{right} - \text{left}$

$$= \tfrac{1}{2}y^2 - 0 = \tfrac{1}{2}y^2$$

82. $L = \text{right} - \text{left}$

$$= 2 - \sqrt[3]{2y}$$

83. $L = \text{right} - \text{left}$

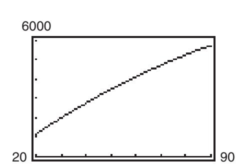
$$= 4 - y^2$$

84. $L = \text{right} - \text{left}$

$$\begin{aligned} &= \frac{2}{y} - 0 \\ &= \frac{2}{y} \end{aligned}$$

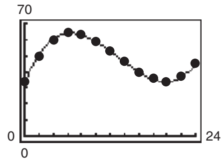
85. $L = -0.294x^2 + 97.744x - 664.875,\ 20 \le x \le 90$

(a)



(b) $L = 2000$ when $x \approx 29.9645 \approx 30$ watts.

86. (a)



(b) The model is an excellent fit.

(c) The temperature is increasing from 6 A.M. until noon ($x = 0$ to $x = 6$). Then it decreases until 2 A.M. ($x = 6$ to $x = 20$). Then the temperature increases until 6 A.M. ($x = 20$ to $x = 24$).

(d) The maximum temperature according to the model is about 63.93°F. According to the data, it is 64°F. The minimum temperature according to the model is about 33.98°F. According to the data, it is 34°F.

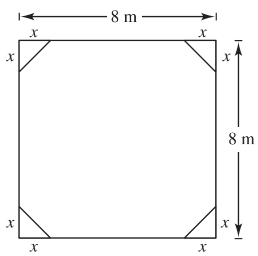
(e) Answers may vary. Temperatures will depend upon the weather patterns, which usually change from day to day.

87. (a) For the average salaries of college professors, a scale of \$10,000 would be appropriate.

(b) For the population of the United States, use a scale of 10,000,000.

(c) For the percent of the civilian workforce that is unemployed, use a scale of 1%.

88.

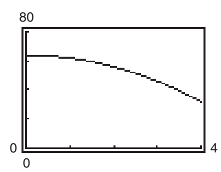


(a) $A = (8)(8) - 4\left(\frac{1}{2}\right)(x)(x)$

$= 64 - 2x^2$

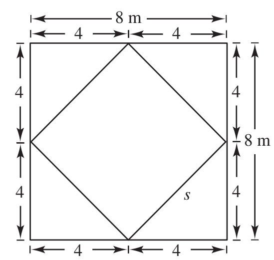
Domain: $0 \le x \le 4$

(b)



Range: $32 \le A \le 64$

(c) When $x = 4$, the resulting figure is a square.

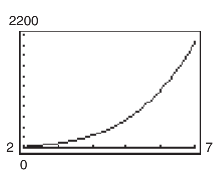


By the Pythagorean Theorem,

$4^2 + 4^2 = s^2 \implies s = \sqrt{32} = 4\sqrt{2}$ meters.

89. $r = 15.639t^3 - 104.75t^2 + 303.5t - 301,\ 2 \le t \le 7$

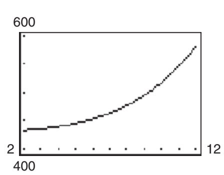
(a)



(b) $\dfrac{r(7) - r(2)}{7 - 2} = \dfrac{2054.927 - 12.112}{5} = 408.563$

The average rate of change from 2002 to 2007 is \$408.563 billion per year. The estimated revenue is increasing each year at a rapid pace.

90. (a)



(b) The average rate of change from 1992 to 2002:

$$\frac{F(12) - F(2)}{12 - 2} = \frac{580.78 - 433.5}{12 - 2}$$

$$= \frac{147.28}{10} = 14.728$$

The number of foreign students increased at a steady rate of 14.728 thousand students each year.

(c) The five-year period of least average rate of change was 1992 to 1997.

$$\frac{F(7) - F(2)}{7 - 2} = \frac{463.74 - 433.5}{7 - 2} = \frac{30.24}{5} = 6.05$$

The five-year period of greatest increase was 1997 to 2002.

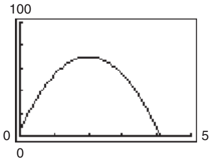
$$\frac{F(12) - F(7)}{12 - 7} = \frac{580.78 - 463.74}{12 - 7} = \frac{117.04}{5} = 23.4$$

The least rate of change was about 6.05 thousand students from 1992 to 1997.

The greatest rate of change was about 23.4 thousand students from 1997 to 2002.

91. $s_0 = 6, v_0 = 64$

(a) $s = -16t^2 + 64t + 6$

(b) 

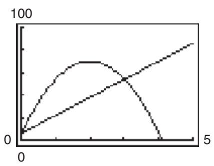
(c) $\dfrac{s(3) - s(0)}{3 - 0} = \dfrac{54 - 6}{3} = 16$

(d) The average rate of change of the height of the object with respect to time over the interval $t_1 = 0$ to $t_2 = 3$ is 16 feet per second.

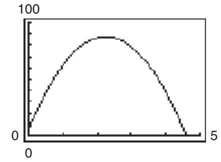
(e) $s(0) = 6, m = 16$

Secant line: $y - 6 = 16(t - 0)$

$$y = 16t + 6$$

(f) 

92. (a) $s = -16t^2 + 72t + 6.5$

(b) 

(c) The average rate of change from $t = 0$ to $t = 4$:

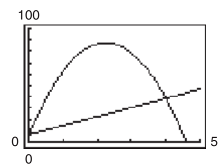
$$\frac{s(4) - s(0)}{4 - 0} = \frac{38.5 - 6.5}{4} = \frac{32}{4} = 8 \text{ feet per second}$$

(d) The slope of the secant line through $(0, s(0))$ and $(4, s(4))$ is positive. The average rate of change of the position of the object from $t = 0$ to $t = 4$ is 8 feet per second.

(e) The equation of the secant line:

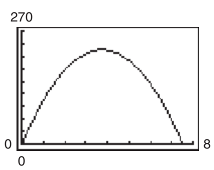
$m = 8,\ y = 8t + 6.5$

(f) The graph is shown in (b).



93. $v_0 = 120, s_0 = 0$

(a) $s = -16t^2 + 120t$

(b) 

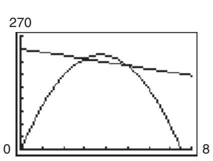
(c) $\dfrac{s(5) - s(3)}{5 - 3} = \dfrac{200 - 216}{2} = -8$

(d) The average decrease in the height of the object over the interval $t_1 = 3$ to $t_2 = 5$ is 8 feet per second.

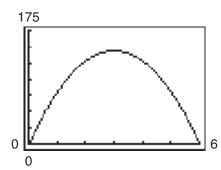
(e) $s(5) = 200, m = -8$

Secant line: $y - 200 = -8(t - 5)$

$$y = -8t + 240$$

(f) 

94. (a) $s = -16t^2 + 96t$

(b) 

(c) The average rate of change from $t = 2$ to $t = 5$:

$$\frac{s(5) - s(2)}{5 - 2} = \frac{80 - 128}{3}$$

$$= -\frac{48}{3} = -16 \text{ feet per second}$$

(d) The slope of the secant line through $(2, s(2))$ and $(5, s(5))$ is negative. The average rate of change of the position of the object from $t = 2$ to $t = 5$ is -16 feet per second.

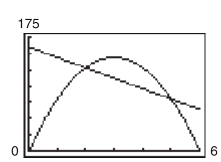
(e) The equation of the secant line: $m = -16$

Using $(2, s(2)) = (2, 128)$ we have

$y - 128 = -16(t - 2)$

$y = -16t + 160.$

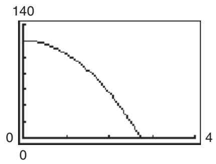
(f) The graph is shown in (b).



95. $v_0 = 0, s_0 = 120$

(a) $s = -16t^2 + 120$

(b)



(c) $\dfrac{s(2) - s(0)}{2 - 0} = \dfrac{56 - 120}{2} = -32$

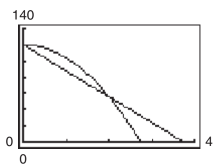
(d) On the interval $t_1 = 0$ to $t_2 = 2$, the height of the object is decreasing at a rate of 32 feet per second.

(e) $s(0) = 120, m = -32$

Secant line: $y - 120 = -32(t - 0)$

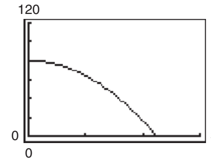
$$y = -32t + 120$$

(f)



96. (a) $s = -16t^2 + 80$

(b)



(c) The average rate of change from $t = 1$ to $t = 2$:

$\dfrac{s(2) - s(1)}{2 - 1} = \dfrac{16 - 64}{1} = -\dfrac{48}{1} = -48$ feet per second

(d) The slope of the secant line through $(1, s(1))$ and $(2, s(2))$ is negative. The average rate of change of the position of the object from $t = 1$ to $t = 2$ is -48 feet per second.

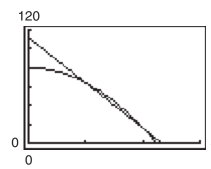
(e) The equation of the tangent line: $m = -48$

Using $(1, s(1)) = (1, 64)$ we have

$y - 64 = -48(t - 1)$

$y = -48t + 112.$

(f) The graph is shown in (b).



97. False. The function $f(x) = \sqrt{x^2 + 1}$ has a domain of all real numbers.

98. False. An odd function is symmetric with respect to the origin, so its domain must include negative values.

99. (a) Even. The graph is a reflection in the x-axis.

(b) Even. The graph is a reflection in the y-axis.

(c) Even. The graph is a vertical translation of f.

(d) Neither. The graph is a horizontal translation of f.

100. Yes, the graph of $x = y^2 + 1$ in Exercise 11 does represent x as a function of y. Each y-value corresponds to only one x-value.

101. $\left(-\frac{3}{2}, 4\right)$

(a) If f is even, another point is $\left(\frac{3}{2}, 4\right)$.

(b) If f is odd, another point is $\left(\frac{3}{2}, -4\right)$.

102. $\left(-\frac{5}{3}, -7\right)$

(a) If f is even, another point is $\left(\frac{5}{3}, -7\right)$.

(b) If f is odd, another point is $\left(\frac{5}{3}, 7\right)$.

103. $(4, 9)$

(a) If f is even, another point is $(-4, 9)$.

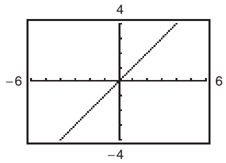
(b) If f is odd, another point is $(-4, -9)$.

104. $(5, -1)$

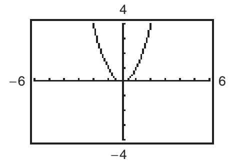
(a) If f is even, another point is $(-5, -1)$.

(b) If f is odd, another point is $(-5, 1)$.

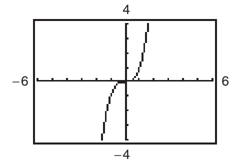
105. (a) $y = x$



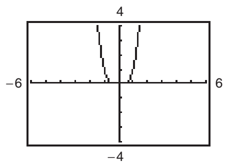
(b) $y = x^2$



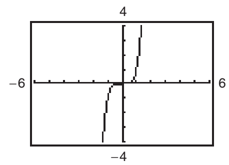
(c) $y = x^3$



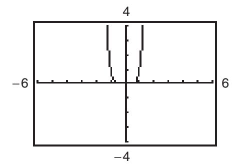
(d) $y = x^4$



(e) $y = x^5$

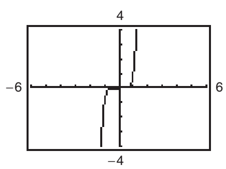


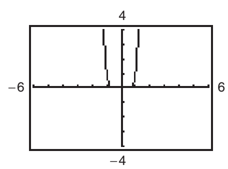
(f) $y = x^6$



All the graphs pass through the origin. The graphs of the odd powers of x are symmetric with respect to the origin and the graphs of the even powers are symmetric with respect to the y-axis. As the powers increase, the graphs become flatter in the interval $-1 < x < 1$.

106. The graph of $y = x^7$ will pass through the origin and will be symmetric with the origin.
The graph of $y = x^8$ will pass through the origin and will be symmetric with respect to the y-axis.





107.
$$\begin{aligned} x^2 - 10x &= 0 \\ x(x - 10) &= 0 \\ x = 0 \quad \text{or} \quad x &= 10 \end{aligned}$$

108.
$$\begin{aligned} 100 - (x - 5)^2 &= 0 \\ (x - 5)^2 &= 100 \\ x - 5 &= \pm 10 \\ x - 5 = -10 &\implies x = -5 \\ x - 5 = 10 &\implies x = 15 \end{aligned}$$

109.
$$\begin{aligned} x^3 - x &= 0 \\ x(x^2 - 1) &= 0 \\ x = 0 \quad \text{or} \quad x^2 - 1 &= 0 \\ x^2 &= 1 \\ x &= \pm 1 \end{aligned}$$

110.
$$\begin{aligned} 16x^2 - 40x + 25 &= 0 \\ (4x - 5)(4x - 5) &= 0 \\ 4x - 5 = 0 &\implies x = \tfrac{5}{4} \end{aligned}$$

111. $f(x) = 5x - 8$

(a) $f(9) = 5(9) - 8 = 37$

(b) $f(-4) = 5(-4) - 8 = -28$

(c) $f(x - 7) = 5(x - 7) - 8 = 5x - 35 - 8 = 5x - 43$

112. $f(x) = x^2 - 10x$

(a) $f(4) = (4)^2 - 10(4) = 16 - 40 = -24$

(b) $f(-8) = (-8)^2 - 10(-8) = 64 + 80 = 144$

(c) $f(x - 4) = (x - 4)^2 - 10(x - 4)$
$= x^2 - 8x + 16 - 10x + 40 = x^2 - 18x + 56$

113. $f(x) = \sqrt{x - 12} - 9$

(a) $f(12) = \sqrt{12 - 12} - 9 = 0 - 9 = -9$

(b) $f(40) = \sqrt{40 - 12} - 9 = \sqrt{28} - 9 = 2\sqrt{7} - 9$

(c) $f(-\sqrt{36})$ does not exist. The given value is not in the domain of the function.

114. $f(x) = x^4 - x - 5$

(a) $f(-1) = (-1)^4 - (-1) - 5 = 1 + 1 - 5 = -3$

(b) $f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^4 - \frac{1}{2} - 5 = -\frac{87}{16}$

(c) $f\left(2\sqrt{3}\right) = \left(2\sqrt{3}\right)^4 - 2\sqrt{3} - 5$

$= 16(9) - 2\sqrt{3} - 5 = 139 - 2\sqrt{3}$

115.
$$
\begin{aligned}
f(x) &= x^2 - 2x + 9\\
f(3 + h) &= (3 + h)^2 - 2(3 + h) + 9\\
&= 9 + 6h + h^2 - 6 - 2h + 9\\
&= h^2 + 4h + 12\\
f(3) &= 3^2 - 2(3) + 9 = 12
\end{aligned}
$$

$$\frac{f(3 + h) - f(3)}{h} = \frac{(h^2 + 4h + 12) - (12)}{h} = \frac{h^2 + 4h}{h} = \frac{h(h + 4)}{h} = h + 4, \quad h \neq 0$$

116. $f(x) = 5 + 6x - x^2$, $\dfrac{f(6 + h) - f(6)}{h}, h \neq 0$

$$
\begin{aligned}
\frac{f(6 + h) - f(6)}{h} &= \frac{5 + 6(6 + h) - (6 + h)^2 - 5 - 6(6) + 6^2}{h}\\
&= \frac{5 + 36 + 6h - 36 - 12h - h^2 - 5 - 36 + 36}{h}\\
&= \frac{-h^2 - 6h}{h} = -h - 6, \quad h \neq 0
\end{aligned}
$$

Section 1.6 A Library of Parent Functions

- You should be able to identify and graph the following types of functions:
  - (a) Linear functions like $f(x) = ax + b$
  - (b) Squaring functions like $f(x) = x^2$
  - (c) Cubic functions like $f(x) = x^3$
  - (d) Square root functions like $f(x) = \sqrt{x}$
  - (e) Reciprocal functions like $f(x) = \dfrac{1}{x}$
  - (f) Constant functions like $f(x) = c$
  - (g) Absolute value functions like $f(x) = |x|$
  - (h) Step and piecewise-defined functions like $f(x) = [\![x]\!]$
- You should be able to determine the following about these parent functions:
  - (a) Domain and range
  - (b) x-intercept(s) and y-intercept
  - (c) Symmetries
  - (d) Where it is increasing, decreasing, or constant
  - (e) If it is odd, even or neither
  - (f) Relative maximums and relative minimums

Vocabulary Check

1. $f(x) = [\![x]\!]$
(g) greatest integer function

2. $f(x) = x$
(i) identity function

3. $f(x) = \frac{1}{x}$
(h) reciprocal function

4. $f(x) = x^2$
(a) squaring function

5. $f(x) = \sqrt{x}$
(b) square root function

6. $f(x) = c$
(e) constant function

7. $f(x) = |x|$
(f) absolute value function

8. $f(x) = x^3$
(c) cubic function

9. $f(x) = ax + b$
(d) linear function

---

1. (a) $f(1) = 4,\ f(0) = 6$

$(1, 4)$ and $(0, 6)$

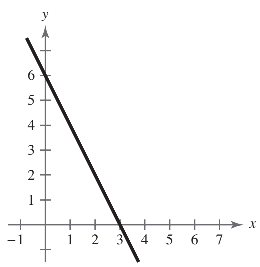
$$m = \frac{6 - 4}{0 - 1} = -2$$

$$y - 6 = -2(x - 0)$$

$$y = -2x + 6$$

$$f(x) = -2x + 6$$

(b)



2. (a) $f(-3) = -8,\ f(1) = 2$

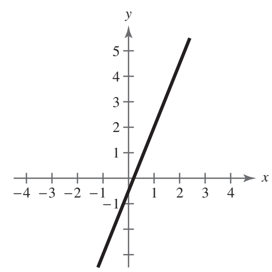
$(-3, -8), (1, 2)$

$$m = \frac{2 - (-8)}{1 - (-3)} = \frac{10}{4} = \frac{5}{2}$$

$$f(x) - 2 = \frac{5}{2}(x - 1)$$

$$f(x) = \frac{5}{2}x - \frac{1}{2}$$

(b)



3. (a) $f(5) = -4,\ f(-2) = 17$

$(5, -4)$ and $(-2, 17)$

$$m = \frac{17 - (-4)}{-2 - 5} = \frac{21}{-7} = -3$$

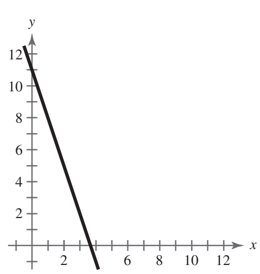
$$y - (-4) = -3(x - 5)$$

$$y + 4 = -3x + 15$$

$$y = -3x + 11$$

$$f(x) = -3x + 11$$

(b)



4. (a) $f(3) = 9,\ f(-1) = -11$

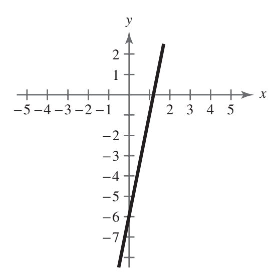
$(3, 9), (-1, -11)$

$$m = \frac{-11 - 9}{-1 - 3} = \frac{-20}{-4} = 5$$

$$f(x) - 9 = 5(x - 3)$$

$$f(x) = 5x - 6$$

(b)



5. (a) $f(-5) = -1,\ f(5) = -1$

$(-5, -1)$ and $(5, -1)$

$$m = \frac{-1 - (-1)}{5 - (-5)} = \frac{0}{10} = 0$$

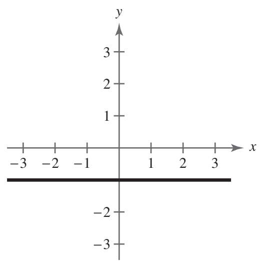
$$y - (-1) = 0(x - (-5))$$

$$y + 1 = 0$$

$$y = -1$$

$$f(x) = -1$$

(b)



6. (a) $f(-10) = 12,\ f(16) = -1$

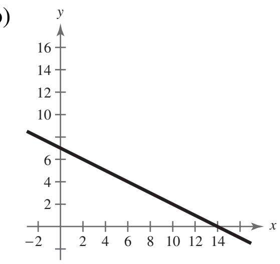
$(-10, 12), (16, -1)$

$$m = \frac{-1 - 12}{16 - (-10)} = \frac{-13}{26} = -\frac{1}{2}$$

$$f(x) - (-1) = -\frac{1}{2}(x - 16)$$

$$f(x) = -\frac{1}{2}x + 7$$

(b)



7. (a) $f\left(\frac{1}{2}\right) = -6, f(4) = -3$

$\left(\frac{1}{2}, -6\right)$ and $(4, -3)$

$$m = \frac{-3 - (-6)}{4 - (1/2)} = \frac{3}{7/2} = \frac{6}{7}$$

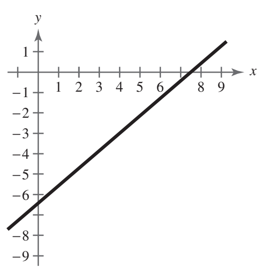
$$y - (-3) = \frac{6}{7}(x - 4)$$

$$y + 3 = \frac{6}{7}x - \frac{24}{7}$$

$$y = \frac{6}{7}x - \frac{45}{7}$$

$$f(x) = \frac{6}{7}x - \frac{45}{7}$$

(b)



8. (a) $f\left(\frac{2}{3}\right) = -\frac{15}{2}, f(-4) = -11$

$\left(\frac{2}{3}, -\frac{15}{2}\right), (-4, -11)$

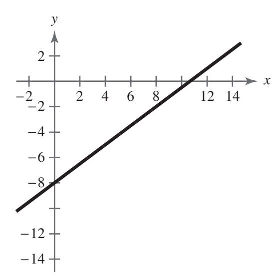
$$m = \frac{-11 - (-15/2)}{-4 - (2/3)}$$

$$= \frac{-7/2}{-14/3} = \left(-\frac{7}{2}\right) \cdot \left(-\frac{3}{14}\right) = \frac{3}{4}$$

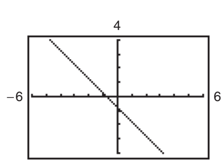
$$f(x) - (-11) = \frac{3}{4}(x - (-4))$$

$$f(x) = \frac{3}{4}x - 8$$

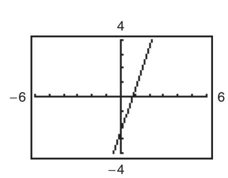
(b)



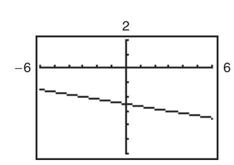
9. $f(x) = -x - \frac{3}{4}$



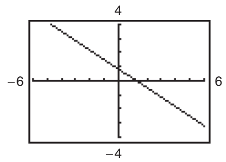
10. $f(x) = 3x - \frac{5}{2}$



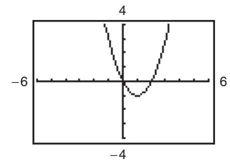
11. $f(x) = -\frac{1}{6}x - \frac{5}{2}$



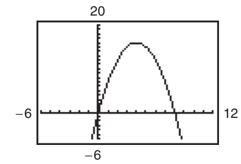
12. $f(x) = \frac{5}{6} - \frac{2}{3}x$



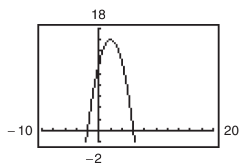
13. $f(x) = x^2 - 2x$



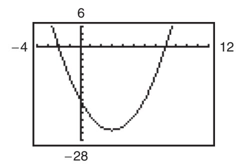
14. $f(x) = -x^2 + 8x$



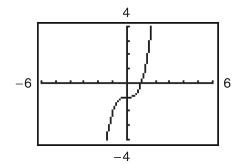
15. $h(x) = -x^2 + 4x + 12$



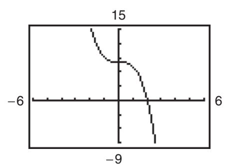
16. $g(x) = x^2 - 6x - 16$



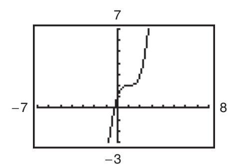
17. $f(x) = x^3 - 1$



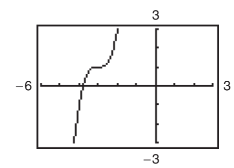
18. $f(x) = 8 - x^3$



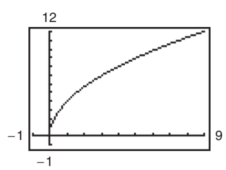
19. $f(x) = (x - 1)^3 + 2$



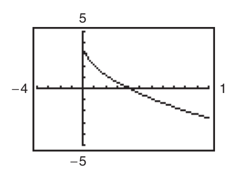
20. $g(x) = 2(x + 3)^3 + 1$



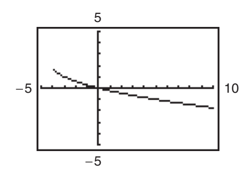
21. $f(x) = 4\sqrt{x}$



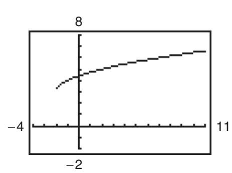
22. $f(x) = 4 - 2\sqrt{x}$



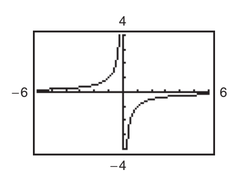
23. $g(x) = 2 - \sqrt{x + 4}$



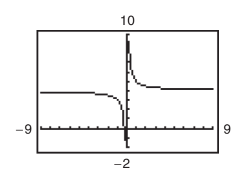
24. $h(x) = \sqrt{x + 2} + 3$



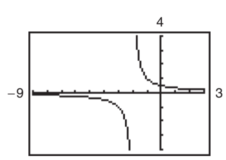
25. $f(x) = -\dfrac{1}{x}$



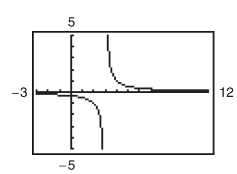
26. $f(x) = 4 + \dfrac{1}{x}$



27. $h(x) = \dfrac{1}{x + 2}$



28. $k(x) = \dfrac{1}{x - 3}$



29. $f(x) = [\![x]\!]$

(a) $f(2.1) = 2$

(b) $f(2.9) = 2$

(c) $f(-3.1) = -4$

(d) $f\left(\dfrac{7}{2}\right) = 3$

30. $g(x) = 2[\![x]\!]$

(a) $g(-3) = 2[\![-3]\!] = 2(-3) = -6$

(b) $g(0.25) = 2[\![0.25]\!] = 2(0) = 0$

(c) $g(9.5) = 2[\![9.5]\!] = 2(9) = 18$

(d) $g\left(\frac{11}{3}\right) = 2\left[\!\!\left[\frac{11}{3}\right]\!\!\right] = 2(3) = 6$

31. $h(x) = [\![x + 3]\!]$

(a) $h(-2) = [\![1]\!] = 1$

(b) $h\left(\frac{1}{2}\right) = [\![3.5]\!] = 3$

(c) $h(4.2) = [\![7.2]\!] = 7$

(d) $h(-21.6) = [\![-18.6]\!] = -19$

32. $f(x) = 4[\![x]\!] + 7$

(a) $f(0) = 4[\![0]\!] + 7 = 4(0) + 7 = 7$

(b) $f(-1.5) = 4[\![-1.5]\!] + 7 = 4(-2) + 7 = -1$

(c) $f(6) = 4[\![6]\!] + 7 = 4(6) + 7 = 31$

(d) $f\left(\frac{5}{3}\right) = 4\left[\!\!\left[\frac{5}{3}\right]\!\!\right] + 7 = 4(1) + 7 = 11$

33. $h(x) = [\![3x - 1]\!]$

(a) $h(2.5) = [\![6.5]\!] = 6$

(b) $h(-3.2) = [\![-10.6]\!] = -11$

(c) $h\left(\frac{7}{3}\right) = [\![6]\!] = 6$

(d) $h\left(-\frac{21}{3}\right) = [\![-22]\!] = -22$

34. $k(x) = \left[\!\!\left[\frac{1}{2}x + 6\right]\!\!\right]$

(a) $k(5) = \left[\!\!\left[\frac{1}{2}(5) + 6\right]\!\!\right] = [\![8.5]\!] = 8$

(b) $k(-6.1) = \left[\!\!\left[\frac{1}{2}(-6.1) + 6\right]\!\!\right] = [\![2.95]\!] = 2$

(c) $k(0.1) = \left[\!\!\left[\frac{1}{2}(0.1) + 6\right]\!\!\right] = [\![6.05]\!] = 6$

(d) $k(15) = \left[\!\!\left[\frac{1}{2}(15) + 6\right]\!\!\right] = [\![13.5]\!] = 13$

35. $g(x) = 3[\![x - 2]\!] + 5$

(a) $g(-2.7) = 3[\![-4.7]\!] + 5 = 3(-5) + 5 = -10$

(b) $g(-1) = 3[\![-3]\!] + 5 = 3(-3) + 5 = -4$

(c) $g(0.8) = 3[\![-1.2]\!] + 5 = 3(-2) + 5 = -1$

(d) $g(14.5) = 3[\![12.5]\!] + 5 = 3(12) + 5 = 41$

36. $g(x) = -7[\![x + 4]\!] + 6$

(a) $g\left(\frac{1}{8}\right) = -7\left[\!\!\left[\frac{1}{8} + 4\right]\!\!\right] + 6$

$= -7\left[\!\!\left[4\frac{1}{8}\right]\!\!\right] + 6 = -7(4) + 6 = -22$

(b) $g(9) = -7[\![9 + 4]\!] + 6$

$= -7[\![13]\!] + 6 = -7(13) + 6 = -85$

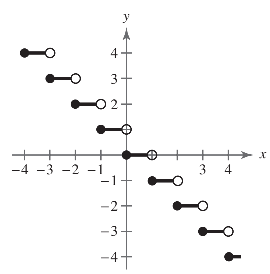
(c) $g(-4) = -7[\![-4 + 4]\!] + 6$

$= -7[\![0]\!] + 6 = -7(0) + 6 = 6$

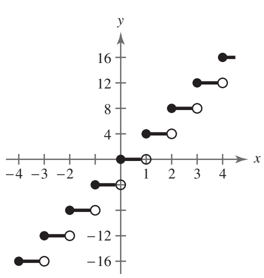
(d) $g\left(\frac{3}{2}\right) = -7\left[\!\!\left[\frac{3}{2} + 4\right]\!\!\right] + 6$

$= -7\left[\!\!\left[5\frac{1}{2}\right]\!\!\right] + 6 = -7(5) + 6 = -29$

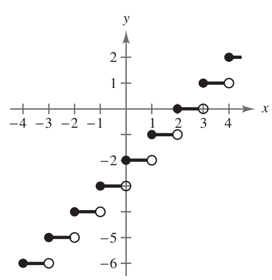
37. $g(x) = -[\![x]\!]$



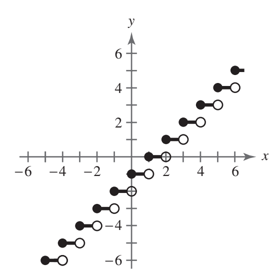
38. $g(x) = 4[\![x]\!]$



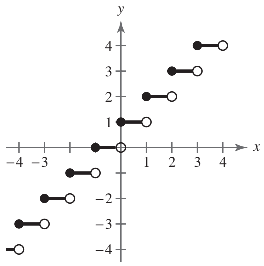
39. $g(x) = [\![x]\!] - 2$



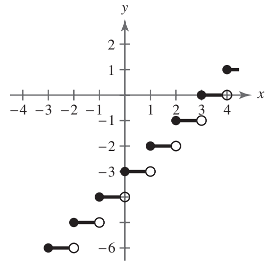
40. $g(x) = [\![x]\!] - 1$



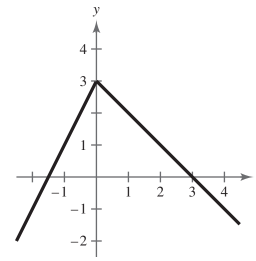
41. $g(x) = [\![x + 1]\!]$



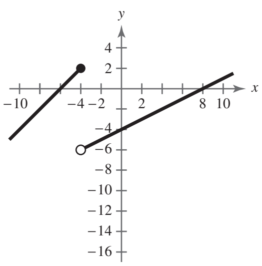
42. $g(x) = [\![x - 3]\!]$



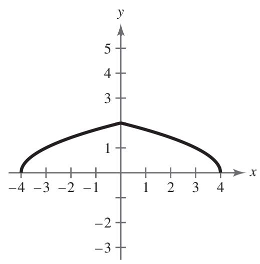
43. $f(x) = \begin{cases} 2x + 3, & x < 0 \\ 3 - x, & x \geq 0 \end{cases}$



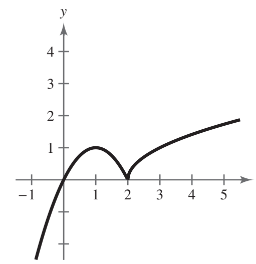
44. $g(x) = \begin{cases} x + 6, & x \leq -4 \\ \frac{1}{2}x - 4, & x > -4 \end{cases}$



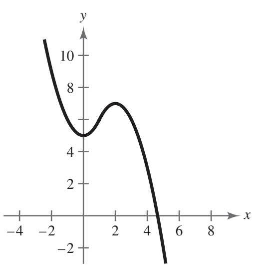
45. $f(x) = \begin{cases} \sqrt{4 + x}, & x < 0 \\ \sqrt{4 - x}, & x \geq 0 \end{cases}$



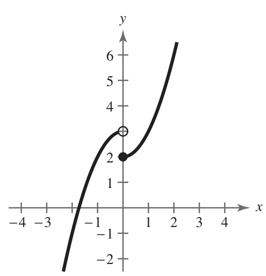
46. $f(x) = \begin{cases} 1 - (x - 1)^2, & x \leq 2 \\ \sqrt{x - 2}, & x > 2 \end{cases}$



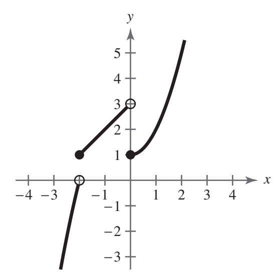
47. $f(x) = \begin{cases} x^2 + 5, & x \leq 1 \\ -x^2 + 4x + 3, & x > 1 \end{cases}$



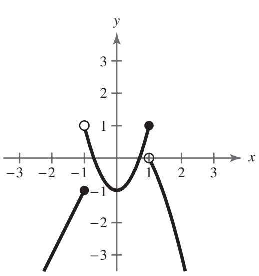
48. $h(x) = \begin{cases} 3 - x^2, & x < 0 \\ x^2 + 2, & x \geq 0 \end{cases}$



49. $h(x) = \begin{cases} 4 - x^2, & x < -2 \\ 3 + x, & -2 \leq x < 0 \\ x^2 + 1, & x \geq 0 \end{cases}$

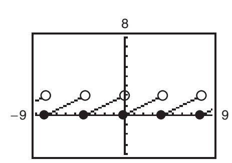


50. $k(x) = \begin{cases} 2x + 1, & x \leq -1 \\ 2x^2 - 1, & -1 < x \leq 1 \\ 1 - x^2, & x1 \end{cases}$



51. $s(x) = 2\left(\frac{1}{4}x - [\![\frac{1}{4}x]\!]\right)$

(a)

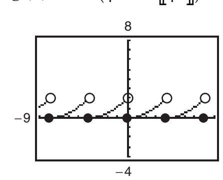


(b) Domain: $(-\infty, \infty)$

Range: $[0, 2)$

(c) Sawtooth pattern

52. (a) $g(x) = 2\left(\frac{1}{4}x - [\![\frac{1}{4}x]\!]\right)^2$



(b) Domain: $(-\infty, \infty)$

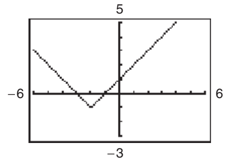
Range: $[0, 2)$

(c) Sawtooth pattern

53. (a) Parent function: $f(x) = |x|$

(b) $g(x) = |x + 2| - 1$

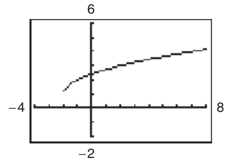
(c)



54. (a) Parent function: $y = \sqrt{x}$

(b) $y = 1 + \sqrt{x + 2}$

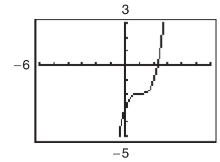
(c)



55. (a) Parent function: $f(x) = x^3$

(b) $g(x) = (x - 1)^3 - 2$

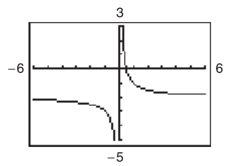
(c)



56. (a) Parent function: $y = \dfrac{1}{x}$

(b) $y = \dfrac{1}{x} - 2$

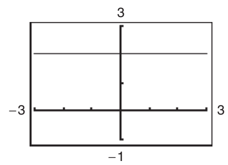
(c)



57. (a) Parent function: $f(x) = c$

(b) $g(x) = 2$

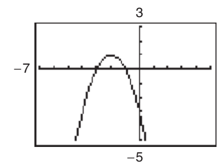
(c)



58. (a) Parent function: $y = x^2$

(b) $y = 1 - (x + 2)^2$

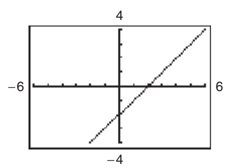
(c)



59. (a) Parent function: $f(x) = x$

(b) $g(x) = x - 2$

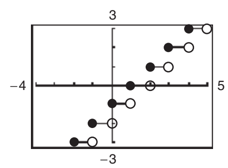
(c)



60. (a) Parent function: $y = [\![x]\!]$

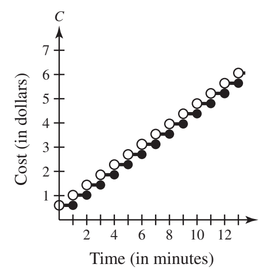
(b) $y = [\![x - 1]\!]$

(c)



61. $C = 0.60 - 0.42[\![1 - t]\!], t > 0$

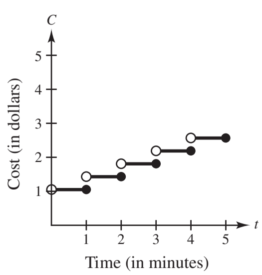
(a)



(b) $C(12.5) = \$5.64$

62. (a) $C_2(t) = 1.05 - 0.38[\![-(t - 1)]\!]$ is the appropriate model since the cost does not increase until after the next minute of conversation has started.

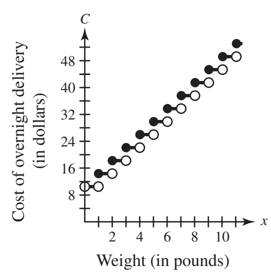
(b)



$C = 1.05 - 0.38[\![-17.75]\!] = \$7.89$

63. $C = 10.75 + 3.95[\![x]\!], \quad x > 0$

(a)



(b) $C(10.33) = 10.75 + 3.95(10) = \$50.25$

64. (a) Model: (Total cost) = (Flat rate) + (Rate per pound)

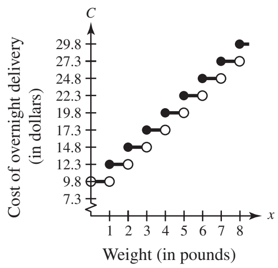
Labels: Total cost $= C$

Flat rate $= 9.80$

Rate per pound $= 2.50[\![x]\!],\ x > 0$

Equation: $C = 9.80 + 2.50[\![x]\!],\ x > 0$

(b)



65. $W(h) = \begin{cases} 12h, & 0 < h \le 40 \\ 18(h - 40) + 480, & h > 40 \end{cases}$

(a) $W(30) = 12(30) = \$360$

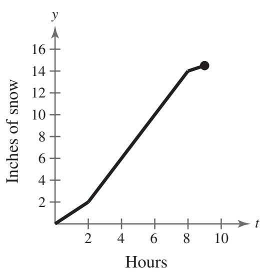
$W(40) = 12(40) = \$480$

$W(45) = 18(5) + 480 = \$570$

$W(50) = 18(10) + 480 = \$660$

(b) $W(h) = \begin{cases} 12h, & 0 < h \le 45 \\ 18(h - 45) + 540, & h > 45 \end{cases}$

66. For the first two hours the slope is 1. For the next six hours, the slope is 2. For the final hour, the slope is $\frac{1}{2}$.



$$f(t) = \begin{cases} t, & 0 \le t \le 2 \\ 2t - 2, & 2 < t \le 8 \\ \frac{1}{2}t + 10, & 8 < t \le 9 \end{cases}$$

To find $f(t) = 2t - 2$, use $m = 2$ and $(2, 2)$.

$y - 2 = 2(t - 2) \implies y = 2t - 2$

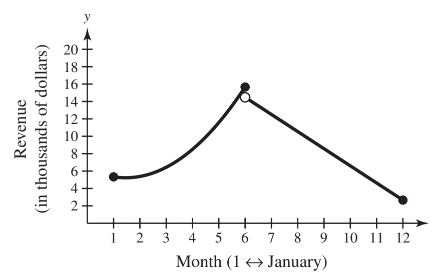
To find $f(t) = \frac{1}{2}t + 10$, use $m = \frac{1}{2}$ and $(8, 14)$.

$y - 14 = \frac{1}{2}(t - 8) \implies y = \frac{1}{2}t + 10$

Total accumulation = 14.5 inches

67. (a) The domain of $f(x) = -1.97x + 26.3$ is $6 < x \le 12$. One way to see this is to notice that this is the equation of a line with negative slope, so the function values are decreasing as x increases, which matches the data for the corresponding part of the table. The domain of $f(x) = 0.505x^2 - 1.47x + 6.3$ is then $1 \le x \le 6$.

(b)



(c) $f(5) = 0.505(5)^2 - 1.47(5) + 6.3$

$= 0.505(25) - 7.35 + 6.3 = 11.575$

$f(11) = -1.97(11) + 26.3 = 4.63$

These values represent the income in thousands of dollars for the months of May and November, respectively.

(d) The model values are very close to the actual values.

Month, x	1	2	3	4	5	6	7	8	9	10	11	12
Revenue, y	5.2	5.6	6.6	8.3	11.5	15.8	12.8	10.1	8.6	6.9	4.5	2.7
Model, $f(x)$	5.3	5.4	6.4	8.5	11.6	15.7	12.5	10.5	8.6	6.6	4.6	2.7

68.

Interval	Intake Pipe	Drainpipe 1	Drainpipe 2
$[0, 5]$	Open	Closed	Closed
$[5, 10]$	Open	Open	Closed
$[10, 20]$	Closed	Closed	Closed
$[20, 30]$	Closed	Closed	Open
$[30, 40]$	Open	Open	Open
$[40, 45]$	Open	Closed	Open
$[45, 50]$	Open	Open	Open
$[50, 60]$	Open	Open	Closed

69. False. A piecewise-defined function is a function that is defined by two or more equations over a specified domain. That domain may or may not include x- and y-intercepts.

70. True. $f(x) = 2[\![x]\!]$, $1 \le x < 4$ is equivalent to the given piecewise function.

71. For the line through $(0, 6)$ and $(3, 2)$: $m = \dfrac{6 - 2}{0 - 3} = -\dfrac{4}{3}$

$$y - 6 = -\frac{4}{3}(x - 0) \implies y = -\frac{4}{3}x + 6$$

For the line through $(3, 2)$ and $(8, 0)$: $m = \dfrac{2 - 0}{3 - 8} = -\dfrac{2}{5}$

$$y - 0 = -\frac{2}{5}(x - 8) \implies y = -\frac{2}{5}x + \frac{16}{5}$$

$$f(x) = \begin{cases} -\frac{4}{3}x + 6, & 0 \le x \le 3 \\ -\frac{2}{5}x + \frac{16}{5}, & 3 < x \le 8 \end{cases}$$

Note that the respective domains can also be $0 \le x < 3$ and $3 \le x \le 8$.

72. $f(x) = \begin{cases} x^2, & x \le 2 \\ 7 - x, & x > 2 \end{cases}$

73. $3x + 4 \le 12 - 5x$

$8x + 4 \le 12$

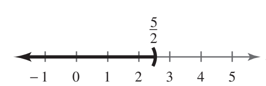
$8x \le 8$

$x \le 1$

74. $2x + 1 > 6x - 9$

$10 > 4x$

$\frac{5}{2} > x$ or $x < \frac{5}{2}$



75. L_1: $(-2, -2)$ and $(2, 10)$

$$m_1 = \frac{10 - (-2)}{2 - (-2)} = \frac{12}{4} = 3$$

L_2: $(-1, 3)$ and $(3, 9)$

$$m_2 = \frac{9 - 3}{3 - (-1)} = \frac{6}{4} = \frac{3}{2}$$

The lines are neither parallel nor perpendicular.

76. L_1: $(-1, -7)$, $(4, 3)$

$$m_1 = \frac{3 - (-7)}{4 - (-1)} = \frac{10}{5} = 2$$

L_2: $(1, 5)$, $(-2, -7)$

$$m_2 = \frac{5 - (-7)}{1 - (-2)} = \frac{12}{3} = 4$$

Because the slopes are neither the same nor negative reciprocals, the lines L_1 and L_2 are neither parallel nor perpendicular.

Section 1.7 Transformations of Functions

- You should know the basic types of transformations.

  Let $y = f(x)$ and let c be a positive real number.

1. $h(x) = f(x) + c$	Vertical shift c units upward
2. $h(x) = f(x) - c$	Vertical shift c units downward
3. $h(x) = f(x - c)$	Horizontal shift c units to the right
4. $h(x) = f(x + c)$	Horizontal shift c units to the left
5. $h(x) = -f(x)$	Reflection in the x-axis
6. $h(x) = f(-x)$	Reflection in the y-axis
7. $h(x) = cf(x), c > 1$	Vertical stretch
8. $h(x) = cf(x), 0 < c < 1$	Vertical shrink
9. $h(x) = f(cx), c > 1$	Horizontal shrink
10. $h(x) = f(cx), 0 < c < 1$	Horizontal stretch

Vocabulary Check

1. rigid **2.** $-f(x)$; $f(-x)$ **3.** nonrigid

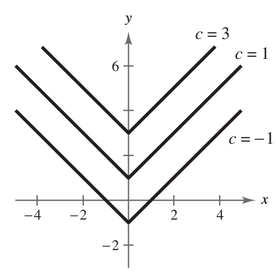
4. horizontal shrink; horizontal stretch **5.** vertical stretch; vertical shrink **6.** (a) iv (b) ii (c) iii (d) i

1. (a) $f(x) = |x| + c$ Vertical shifts

$c = -1 : f(x) = |x| - 1$ 1 unit down

$c = 1 : f(x) = |x| + 1$ 1 unit up

$c = 3 : f(x) = |x| + 3$ 3 units up

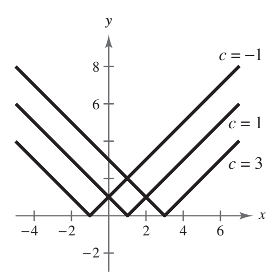


(b) $f(x) = |x - c|$ Horizontal shifts

$c = -1 : f(x) = |x + 1|$ 1 unit left

$c = 1 : f(x) = |x - 1|$ 1 unit right

$c = 3 : f(x) = |x - 3|$ 3 units right

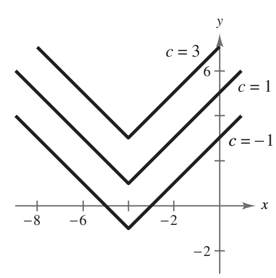


(c) $f(x) = |x + 4| + c$ Horizontal shift four units left and a vertical shift

$c = -1 : f(x) = |x + 4| - 1$ 1 unit down

$c = 1 : f(x) = |x + 4| + 1$ 1 unit up

$c = 3 : f(x) = |x + 4| + 3$ 3 units up



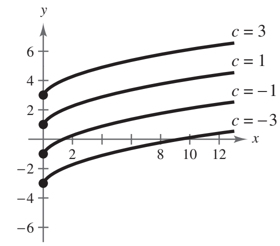
2. (a) $f(x) = \sqrt{x} + c$ Vertical shifts

$c = -3 : f(x) = \sqrt{x} - 3$ 3 units down

$c = -1 : f(x) = \sqrt{x} - 1$ 1 unit down

$c = 1 : f(x) = \sqrt{x} + 1$ 1 unit up

$c = 3 : f(x) = \sqrt{x} + 3$ 3 units up



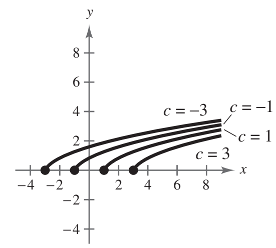
(b) $f(x) = \sqrt{x - c}$ Horizontal shifts

$c = -3 : f(x) = \sqrt{x + 3}$ 3 units left

$c = -1 : f(x) = \sqrt{x + 1}$ 1 unit left

$c = 1 : f(x) = \sqrt{x - 1}$ 1 unit right

$c = 3 : f(x) = \sqrt{x - 3}$ 3 units right



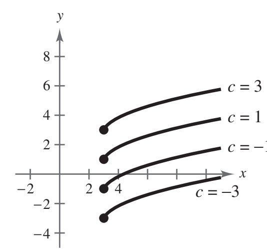
(c) $f(x) = \sqrt{x - 3} + c$ Horizontal shift 3 units right and a vertical shift

$c = -3 : f(x) = \sqrt{x - 3} - 3$ 3 units down

$c = -1 : f(x) = \sqrt{x - 3} - 1$ 1 unit down

$c = 1 : f(x) = \sqrt{x - 3} + 1$ 1 unit up

$c = 3 : f(x) = \sqrt{x - 3} + 3$ 3 units up

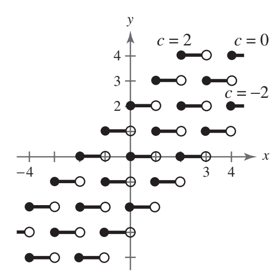


3. (a) $f(x) = \llbracket x \rrbracket + c$ Vertical shifts

$c = -2 : f(x) = \llbracket x \rrbracket - 2$ 2 units down

$c = 0 : f(x) = \llbracket x \rrbracket$ Parent function

$c = 2 : f(x) = \llbracket x \rrbracket + 2$ 2 units up

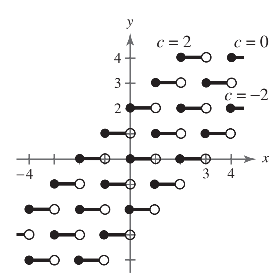


(b) $f(x) = \llbracket x + c \rrbracket$ Horizontal shifts

$c = -2 : f(x) = \llbracket x - 2 \rrbracket$ 2 units right

$c = 0 : f(x) = \llbracket x \rrbracket$ Parent function

$c = 2 : f(x) = \llbracket x + 2 \rrbracket$ 2 units left

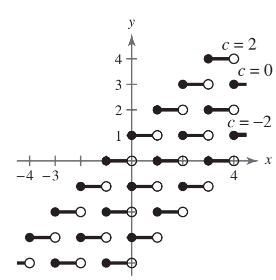


(c) $f(x) = \llbracket x - 1 \rrbracket + c$ Horizontal shift 1 unit right and a vertical shift

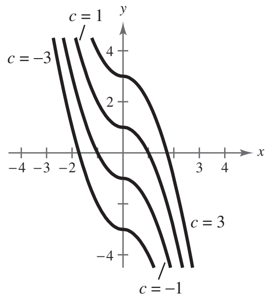
$c = -2 : f(x) = \llbracket x - 1 \rrbracket - 2$ 2 units down

$c = 0 : f(x) = \llbracket x - 1 \rrbracket$

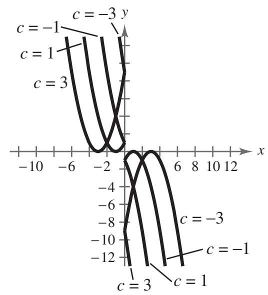
$c = 2 : f(x) = \llbracket x - 1 \rrbracket + 2$ 2 units up



4. (a) $f(x) = \begin{cases} x^2 + c, & x < 0 \\ -x^2 + c, & x \geq 0 \end{cases}$

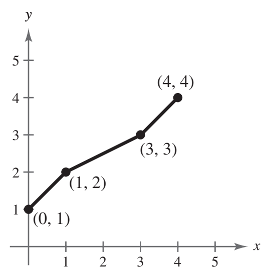


(b) $f(x) = \begin{cases} (x + c)^2, & x < 0 \\ -(x + c)^2, & x \geq 0 \end{cases}$



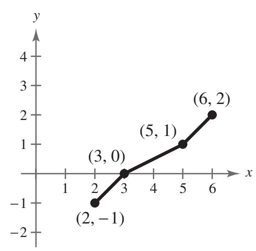
5. (a) $y = f(x) + 2$

Vertical shift 2 units upward



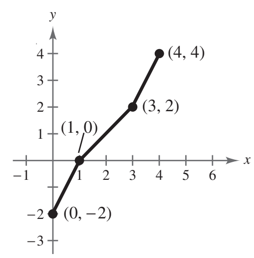
(b) $y = f(x - 2)$

Horizontal shift 2 units to the right



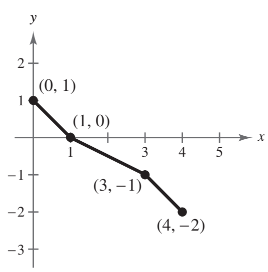
(c) $y = 2f(x)$

Vertical stretch (each y-value is multiplied by 2)



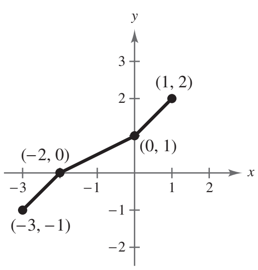
(d) $y = -f(x)$

Reflection in the x-axis



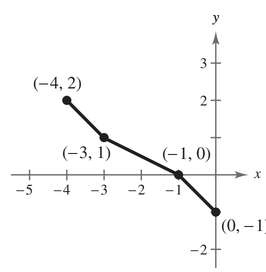
(e) $y = f(x + 3)$

Horizontal shift 3 units to the left



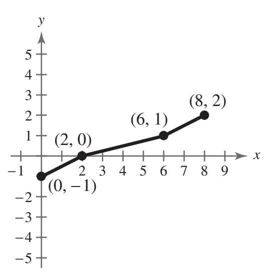
(f) $y = f(-x)$

Reflection in the y-axis



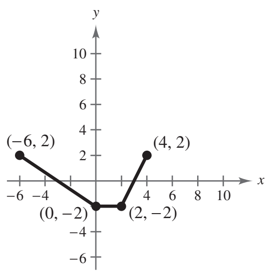
(g) $y = f\left(\frac{1}{2}x\right)$

Horizontal stretch (each x-value is multiplied by 2)



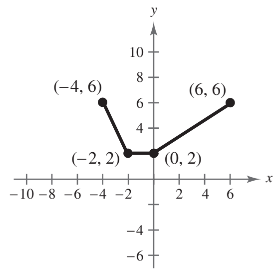
6. (a) $y = f(-x)$

Reflection in the y-axis



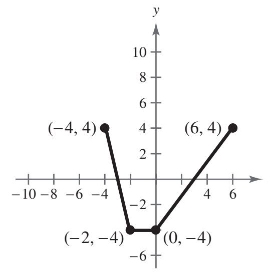
(b) $y = f(x) + 4$

Vertical shift 4 units upward



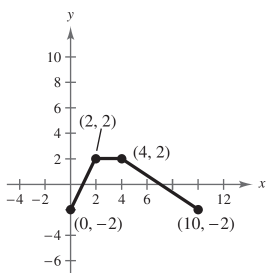
(c) $y = 2f(x)$

Vertical stretch (each y-value is multiplied by 2)



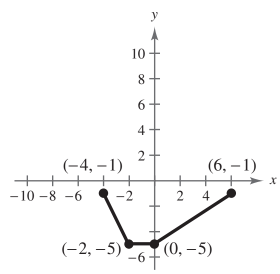
(d) $y = -f(x - 4)$

Reflection in the x-axis and a horizontal shift 4 units to the right



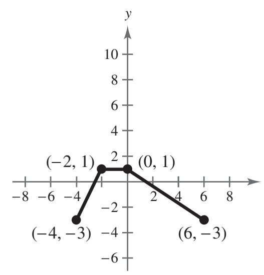
(e) $y = f(x) - 3$

Vertical shift 3 units downward



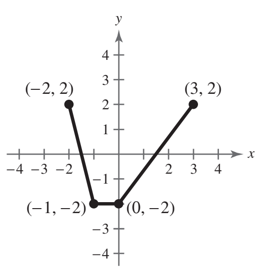
(f) $y = -f(x) - 1$

Reflection in the x-axis and a vertical shift 1 unit downward



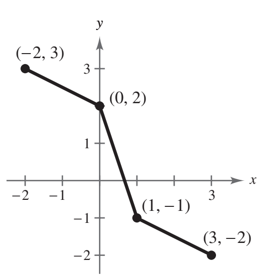
(g) $y = f(2x)$

Horizontal shrink (each x-value is divided by 2)



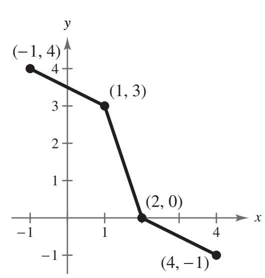
7. (a) $y = f(x) - 1$

Vertical shift 1 unit downward



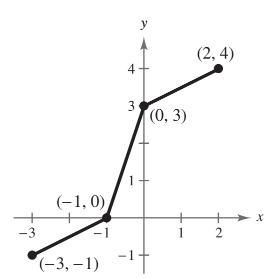
(b) $y = f(x - 1)$

Horizontal shift 1 unit to the right



(c) $y = f(-x)$

Reflection about the y-axis



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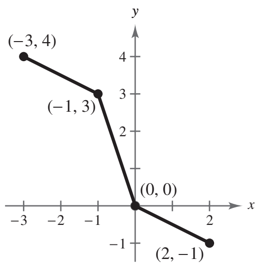
7. —CONTINUED—

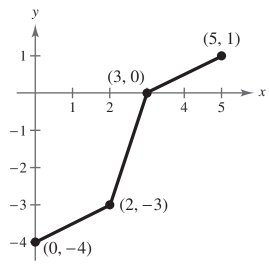
(d) $y = f(x + 1)$

Horizontal shift 1 unit to the left



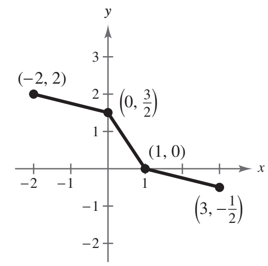
(e) $y = -f(x - 2)$

Reflection about the x-axis and a horizontal shift 2 units to the right



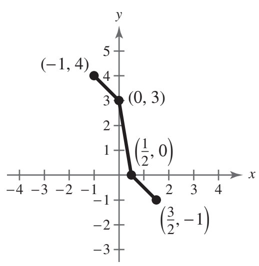
(f) $y = \frac{1}{2}f(x)$

Vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{2}\right)$



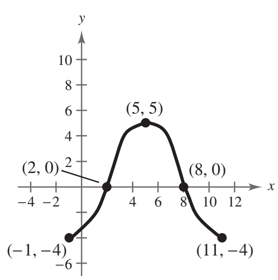
(g) $y = f(2x)$

Horizontal shrink $\left(\text{each } x\text{-value is multiplied by } \frac{1}{2}\right)$



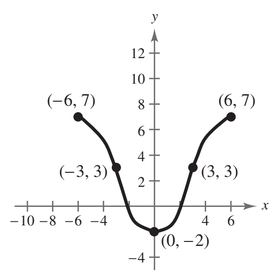
8. (a) $y = f(x - 5)$

Horizontal shift 5 units to the right



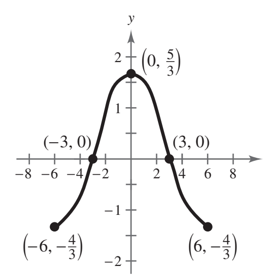
(b) $y = -f(x) + 3$

Reflection in the x-axis and a vertical shift 3 units upward



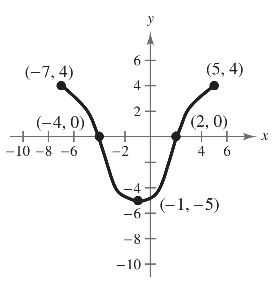
(c) $y = \frac{1}{3}f(x)$

Vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{3}\right)$



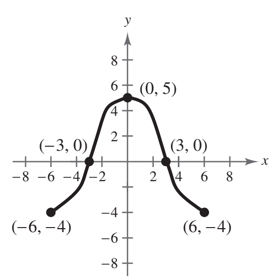
(d) $y = -f(x + 1)$

Reflection in the x-axis and a horizontal shift 1 unit to the left



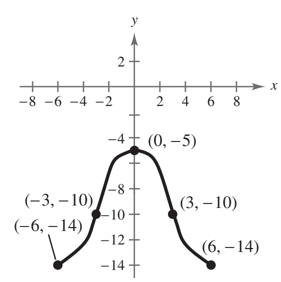
(e) $y = f(-x)$

Reflection in the y-axis



(f) $y = f(x) - 10$

Vertical shift 10 units downward



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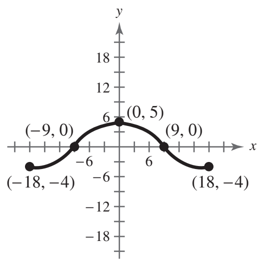
8. —CONTINUED—

(g) $y = f\left(\frac{1}{3}x\right)$

Horizontal stretch (each x-value is multiplied by 3)



9. Parent function: $f(x) = x^2$

(a) Vertical shift 1 unit downward

$g(x) = x^2 - 1$

(b) Reflection about the x-axis, horizontal shift 1 unit to the left, and a vertical shift 1 unit upward

$g(x) = -(x + 1)^2 + 1$

(c) Reflection about the x-axis, horizontal shift 2 units to the right, and a vertical shift 6 units upward

$g(x) = -(x - 2)^2 + 6$

(d) Horizontal shift 5 units to the right and a vertical shift 3 units downward

$g(x) = (x - 5)^2 - 3$

10. Parent function: $f(x) = x^3$

(a) Reflected in the x-axis and shifted upward 1 unit

$g(x) = -x^3 + 1 = 1 - x^3$

(b) Shifted to the right 1 unit and upward 1 unit

$g(x) = (x - 1)^3 + 1$

(c) Reflected in the x-axis and shifted to the left 3 units and downward 1 unit

$g(x) = -(x + 3)^3 - 1$

(d) Shifted to the right 10 units and downward 4 units

$g(x) = (x - 10)^3 - 4$

11. Parent function: $f(x) = |x|$

(a) Vertical shift 5 units upward

$g(x) = |x| + 5$

(b) Reflection in the x-axis and a horizontal shift 3 units to the left

$g(x) = -|x + 3|$

(c) Horizontal shift 2 units to the right and a vertical shift 4 units downward

$g(x) = |x - 2| - 4$

(d) Reflection in the x-axis, horizontal shift 6 units to the right, and a vertical shift 1 unit downward

$g(x) = -|x - 6| - 1$

12. Parent function: $f(x) = \sqrt{x}$

(a) Shifted down 3 units

$g(x) = \sqrt{x} - 3$

(b) Shifted downward 7 units and to the left 1 unit

$g(x) = \sqrt{x + 1} - 7$

(c) Reflected in the x-axis and shifted to the right 5 units and upward 5 units

$g(x) = -\sqrt{x - 5} + 5$

(d) Reflected about the x- and y-axis and shifted to the right 3 units and downward 4 units

$g(x) = -\sqrt{-x + 3} - 4 = -\sqrt{-(x - 3)} - 4$

13. Parent function: $f(x) = x^3$

Horizontal shift 2 units to the right: $y = (x - 2)^3$

14. Parent function: $y = x$

Transformation: vertical shrink

Formula: $y = \frac{1}{2}x$

15. Parent function: $f(x) = x^2$

Reflection in the x-axis: $y = -x^2$

16. Parent function: $y = [\![x]\!]$

Transformation: vertical shift

Formula: $y = [\![x]\!] + 4$

17. Parent function: $f(x) = \sqrt{x}$

Reflection in the x-axis and a vertical shift 1 unit upward: $y = -\sqrt{x} + 1$

18. Parent function: $y = |x|$

Transformation: horizontal shift

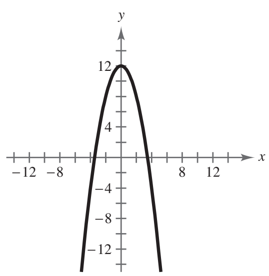
Formula: $y = |x + 2|$

19. $g(x) = 12 - x^2$

(a) Parent function: $f(x) = x^2$

(b) Reflection in the x-axis and a vertical shift 12 units upward

(c)



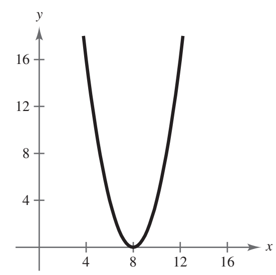
(d) $g(x) = 12 - f(x)$

20. $g(x) = (x - 8)^2$

(a) Parent function: $f(x) = y = x^2$

(b) Horizontal shift of 8 units to the right

(c)



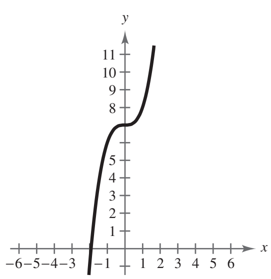
(d) $g(x) = f(x - 8)$

21. $g(x) = x^3 + 7$

(a) Parent function: $f(x) = x^3$

(b) Vertical shift 7 units upward

(c)



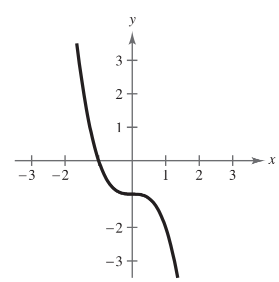
(d) $g(x) = f(x) + 7$

22. $g(x) = -x^3 - 1$

(a) Parent function: $f(x) = x^3$

(b) Reflection in the x-axis; vertical shift of 1 unit downward

(c)



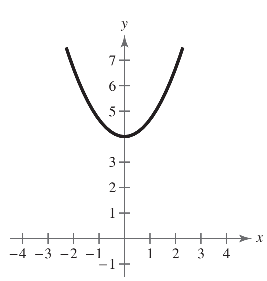
(d) $g(x) = -f(x) - 1$

23. $g(x) = \frac{2}{3}x^2 + 4$

(a) Parent function: $f(x) = x^2$

(b) Vertical shrink of two-thirds, and a vertical shift 4 units upward

(c)



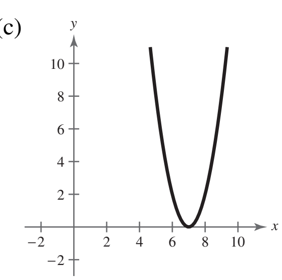
(d) $g(x) = \frac{2}{3}f(x) + 4$

24. $g(x) = 2(x - 7)^2$

(a) Parent function: $f(x) = x^2$

(b) Vertical stretch of 2 and a horizontal shift 7 units to the right of $f(x) = x^2$

(c)



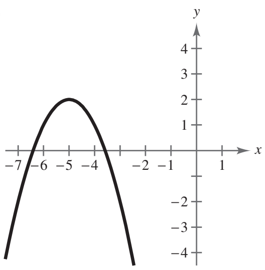
(d) $g(x) = 2f(x - 7)$

25. $g(x) = 2 - (x + 5)^2$

(a) Parent function: $f(x) = x^2$

(b) Reflection in the x-axis, horizontal shift 5 units to the left, and a vertical shift 2 units upward

(c)



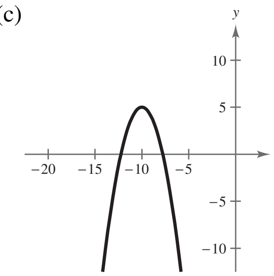
(d) $g(x) = 2 - f(x + 5)$

26. $g(x) = -(x + 10)^2 + 5$

(a) Parent function: $f(x) = x^2$

(b) Reflection in the x-axis; horizontal shift of 10 units to the left; vertical shift of 5 units upward

(c)



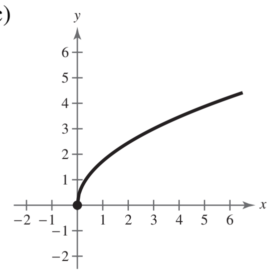
(d) $g(x) = -f(x + 10) + 5$

27. $g(x) = \sqrt{3x}$

(a) Parent function: $f(x) = \sqrt{x}$

(b) Horizontal shrink by $\frac{1}{3}$

(c)



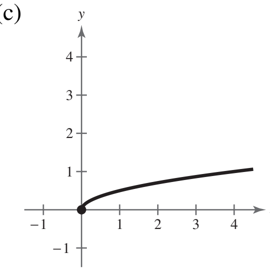
(d) $g(x) = f(3x)$

28. $g(x) = \sqrt{\frac{1}{4}x}$

(a) Parent function: $f(x) = \sqrt{x}$

(b) Horizontal stretch of 4, $f(x) = \sqrt{x}$

(c)



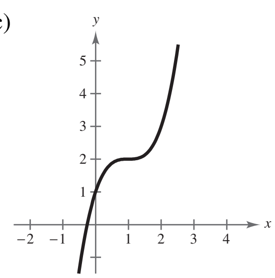
(d) $g(x) = f\left(\frac{1}{4}x\right)$

29. $g(x) = (x - 1)^3 + 2$

(a) Parent function: $f(x) = x^3$

(b) Horizontal shift 1 unit to the right and a vertical shift 2 units upward

(c)



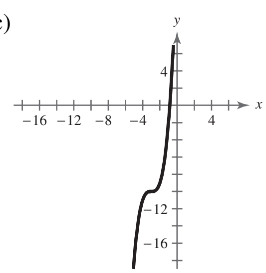
(d) $g(x) = f(x - 1) + 2$

30. $g(x) = (x + 3)^3 - 10$

(a) Parent function: $f(x) = x^3$

(b) Horizontal shift of 3 units to the left; vertical shift of 10 units downward

(c)



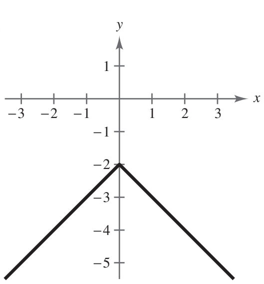
(d) $g(x) = f(x + 3) - 10$

31. $g(x) = -|x| - 2$

(a) Parent function: $f(x) = |x|$

(b) Reflection in the x-axis; vertical shift 2 units downward

(c)



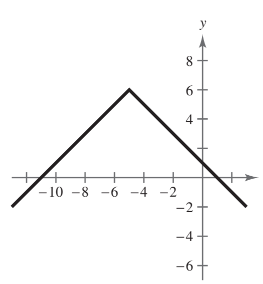
(d) $g(x) = -f(x) - 2$

32. $g(x) = 6 - |x + 5|$

(a) Parent function: $f(x) = |x|$

(b) Reflection in the x-axis; horizontal shift of 5 units to the left; vertical shift of 6 units upward

(c)



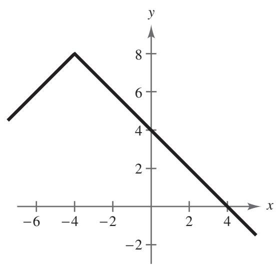
(d) $g(x) = 6 - f(x + 5)$

33. $g(x) = -|x + 4| + 8$

(a) Parent function: $f(x) = |x|$

(b) Reflection in the x-axis, horizontal shift 4 units to the left, and a vertical shift 8 units upward

(c)



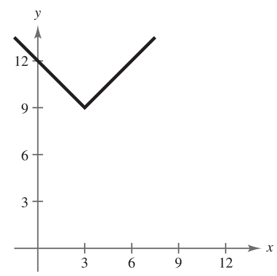
(d) $g(x) = -f(x + 4) + 8$

34. $g(x) = |-x + 3| + 9$

(a) Parent function: $f(x) = |x|$

(b) Reflection in the y-axis; horizontal shift of 3 units to the right; vertical shift of 9 units upward

(c)



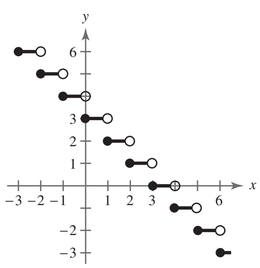
(d) $g(x) = f(-(x - 3)) + 9$

35. $g(x) = 3 - [\![x]\!]$

(a) Parent function: $f(x) = [\![x]\!]$

(b) Reflection in the x-axis and a vertical shift 3 units up

(c)



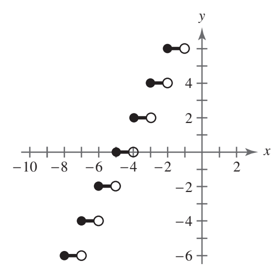
(d) $g(x) = 3 - f(x)$

36. $g(x) = 2[\![x + 5]\!]$

(a) Parent function: $f(x) = [\![x]\!]$

(b) Horizontal shift of 5 units to the left; vertical stretch (each y-value is multiplied by 2)

(c)



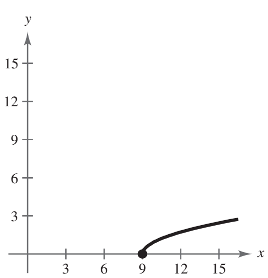
(d) $g(x) = 2f(x + 5)$

37. $g(x) = \sqrt{x - 9}$

(a) Parent function: $f(x) = \sqrt{x}$

(b) Horizontal shift 9 units to the right

(c)



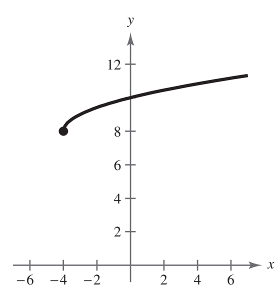
(d) $g(x) = f(x - 9)$

38. $g(x) = \sqrt{x + 4} + 8$

(a) Parent function: $f(x) = \sqrt{x}$

(b) Horizontal shift of 4 units to the left; vertical shift of 8 units upward

(c)



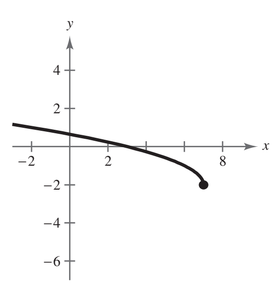
(d) $g(x) = f(x + 4) + 8$

39. $g(x) = \sqrt{7 - x} - 2$ or $g(x) = \sqrt{-(x - 7)} - 2$

(a) Parent function: $f(x) = \sqrt{x}$

(b) Reflection in the y-axis, horizontal shift 7 units to the right, and a vertical shift 2 units downward

(c)



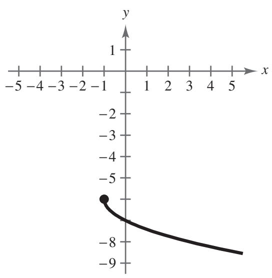
(d) $g(x) = f(7 - x) - 2$

40. $g(x) = -\sqrt{x+1} - 6$

(a) Parent function: $f(x) = \sqrt{x}$

(b) Reflection in the x-axis; horizontal shift of 1 unit to the left; vertical shift of 6 units downward

(c)



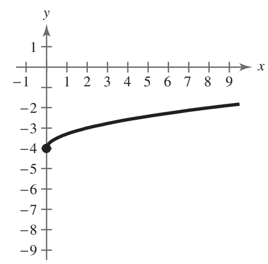
(d) $g(x) = -f(x+1) - 6$

41. $g(x) = \sqrt{\frac{1}{2}x} - 4$

(a) Parent function: $f(x) = \sqrt{x}$

(b) Horizontal stretch (each x-value is multiplied by 2) and a vertical shift 4 units down

(c)



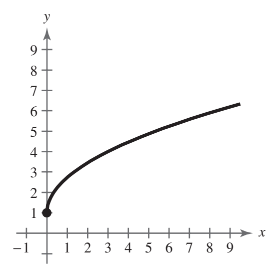
(d) $g(x) = f\left(\frac{1}{2}x\right) - 4$

42. $g(x) = \sqrt{3x} + 1$

(a) Parent function: $f(x) = \sqrt{x}$

(b) Horizontal shrink $\left(\text{each } x\text{-value is multiplied by } \frac{1}{3}\right)$; vertical shift of 1 unit upward

(c)



(d) $g(x) = f(3x) + 1$

43. $f(x) = x^2$ moved 2 units to the right and 8 units down.

$g(x) = (x-2)^2 - 8$

44. $f(x) = x^2$ moved 3 units to the left, 7 units upward, and reflected in the x-axis (in that order)

$g(x) = -(x+3)^2 - 7$

45. $f(x) = x^3$ moved 13 units to the right.

$g(x) = (x-13)^3$

46. $f(x) = x^3$ moved 6 units to the left, 6 units downward, and reflected in the y-axis (in that order)

$g(x) = (-x+6)^3 - 6$ or $g(x) = -(x-6)^3 - 6$

47. $f(x) = |x|$ moved 10 units up and reflected about the x-axis.

$g(x) = -(|x| + 10) = -|x| - 10$

48. $f(x) = |x|$ moved 1 unit to the right and 7 units downward

$g(x) = |x-1| - 7$

49. $f(x) = \sqrt{x}$ moved 6 units to the left and reflected in both the x- and y-axes.

$g(x) = -\sqrt{-x+6}$

50. $f(x) = \sqrt{x}$ moved 9 units downward and reflected in both the x-axis and the y-axis

$g(x) = -\left(\sqrt{-x} - 9\right)$

51. $f(x) = x^2$

(a) Reflection in the x-axis and a vertical stretch (each y-value is multiplied by 3)

$g(x) = -3x^2$

(b) Vertical shift 3 units upward and a vertical stretch (each y-value is multiplied by 4)

$g(x) = 4x^2 + 3$

52. $f(x) = x^3$

(a) Vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{4}\right)$

$g(x) = \frac{1}{4}x^3$

(b) Reflection in the x-axis and a vertical stretch (each y-value is multiplied by 2)

$g(x) = -2x^3$

53. $f(x) = |x|$

(a) Reflection in the x-axis and a vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{2}\right)$

$g(x) = -\frac{1}{2}|x|$

(b) Vertical stretch (each y-value is multiplied by 3) and a vertical shift 3 units downward

$g(x) = 3|x| - 3$

54. $f(x) = \sqrt{x}$

(a) Vertical stretch (each y-value is multiplied by 8)

$g(x) = 8\sqrt{x}$

(b) Reflection in the x-axis and a vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{4}\right)$

$g(x) = -\frac{1}{4}\sqrt{x}$

55. Parent function: $f(x) = x^3$

Vertical stretch (each y-value is multiplied by 2)

$g(x) = 2x^3$

56. Parent function: $f(x) = |x|$

Vertical stretch (each y-value is multiplied by 6)

$g(x) = 6|x|$

57. Parent function: $f(x) = x^2$

Reflection in the x-axis; vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{2}\right)$

$g(x) = -\frac{1}{2}x^2$

58. Parent function: $y = [\![x]\!]$

Horizontal stretch (each x-value is multiplied by 2)

$g(x) = [\![\frac{1}{2}x]\!]$

59. Parent function: $f(x) = \sqrt{x}$

Reflection in the y-axis; vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{2}\right)$

$g(x) = \frac{1}{2}\sqrt{-x}$

60. Parent function: $f(x) = |x|$

Reflection in the x-axis; vertical shift of 2 units downward; vertical stretch (each y-value is multiplied by 2)

$g(x) = -2|x| - 2$

61. Parent function: $f(x) = x^3$

Reflection in the x-axis, horizontal shift 2 units to the right and a vertical shift 2 units upward

$g(x) = -(x - 2)^3 + 2$

62. Parent function: $f(x) = |x|$

Horizontal shift of 4 units to the left and a vertical shift of 2 units downward

$g(x) = |x + 4| - 2$

63. Parent function: $f(x) = \sqrt{x}$

Reflection in the x-axis and a vertical shift 3 units downward

$g(x) = -\sqrt{x} - 3$

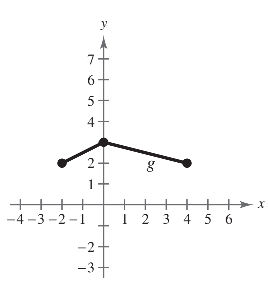
64. Parent function: $f(x) = x^2$

Horizontal shift of 2 units to the right and a vertical shift of 4 units upward.

$g(x) = (x - 2)^2 + 4$

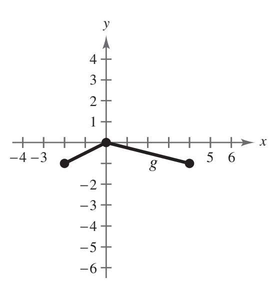
65. (a) $g(x) = f(x) + 2$

Vertical shift 2 units upward



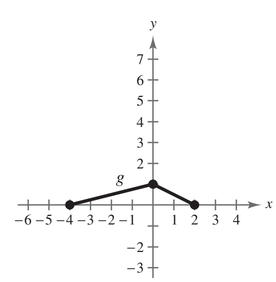
(b) $g(x) = f(x) - 1$

Vertical shift 1 unit downward



(c) $g(x) = f(-x)$

Reflection in the y-axis



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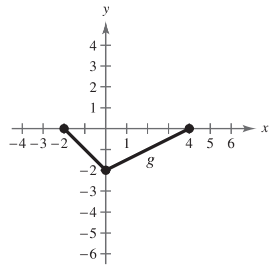
65. —CONTINUED—

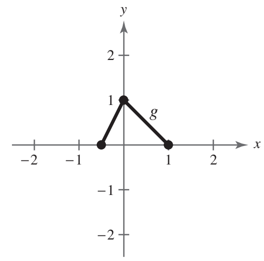
(d) $g(x) = -2f(x)$

Reflection in the x-axis and a vertical stretch (each y-value is multiplied by 2)



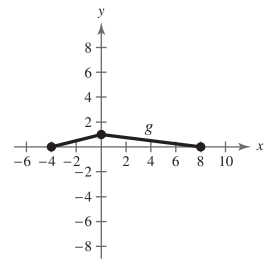
(e) $g(x) = f(4x)$

Horizontal shrink $\left(\text{each } x\text{-value is multiplied by } \frac{1}{4}\right)$



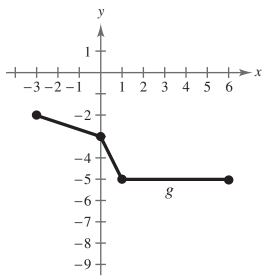
(f) $g(x) = f\left(\frac{1}{2}x\right)$

Horizontal stretch (each x-value is multiplied by 2)



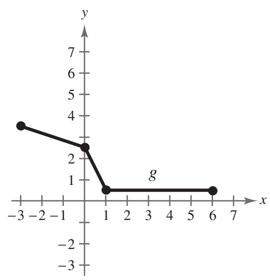
66. (a) $g(x) = f(x) - 5$

Vertical shift 5 units downward



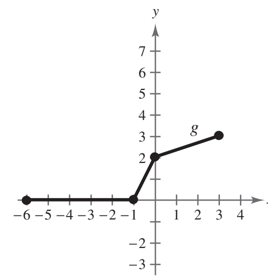
(b) $g(x) = f(x) + \frac{1}{2}$

Vertical shift $\frac{1}{2}$ unit upward



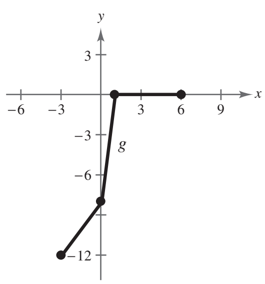
(c) $g(x) = f(-x)$

Reflection in the y-axis



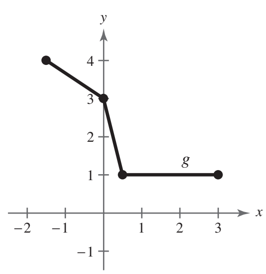
(d) $g(x) = -4f(x)$

Reflection in the x-axis and a vertical stretch (each y-value is multiplied by 4)



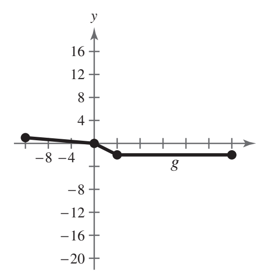
(e) $g(x) = f(2x) + 1$

Horizontal shrink $\left(\text{each } x\text{-value is multiplied by } \frac{1}{2}\right)$ and a vertical shift 1 unit upward



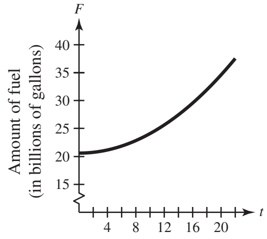
(f) $g(x) = f\left(\frac{1}{4}x\right) - 2$

Horizontal stretch (each x-value is multiplied by 4) and a vertical shift 2 units downward



67. $F = f(t) = 20.6 + 0.035t^2,\ 0 \le t \le 22$

(a) A vertical shrink by 0.035 and a vertical shift of 20.6 units upward



(b) $\dfrac{f(22) - f(0)}{22 - 0} = \dfrac{37.54 - 20.6}{22} = 0.77$

The average increase in fuel used by trucks was 0.77 billion gallons per year between 1980 and 2002.

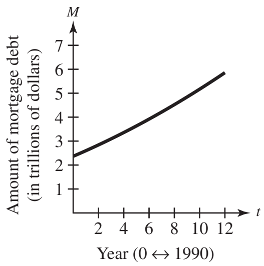
(c) $g(t) = 20.6 + 0.035(t + 10)^2 = f(t + 10)$

This represents a horizontal shift 10 units to the left.

(d) $g(20) = 52.1$ billion gallons

Yes. There are many factors involved here. The number of trucks on the road continues to increase but are more fuel efficient. The availability and the cost of overseas and domestic fuel also plays a role in usage.

68. (a) The graph is a horizontal shift 20.396 units to the left of the graph of the common function $f(x) = x^2$ and a vertical shrink by a factor of 0.0054.



(b) $f(t) = 0.0054(t + 30.396)^2$

By shifting the graph 10 units to the left, you obtain $t = 0$ represents 1990.

69. True, since $|x| = |-x|$, the graphs of $f(x) = |x| + 6$ and $f(x) = |-x| + 6$ are identical.

70. False. The point $(-2, -67)$ lies on the transformation.

71. (a) The profits were only $\frac{3}{4}$ as large as expected:

$g(t) = \frac{3}{4}f(t)$

(b) The profits were \$10,000 greater than predicted:

$g(t) = f(t) + 10{,}000$

(c) There was a two-year delay: $g(t) = f(t - 2)$

72. If you consider the x-axis to be a mirror, each of the y-values of the graph of $y = -f(x)$ is the mirror image of each of the y-values of the graph of $y = f(x)$.

73. $y = f(x + 2) - 1$

Horizontal shift 2 units to the left and a vertical shift 1 unit downward

$(0, 1) \rightarrow (0 - 2, 1 - 1) = (-2, 0)$

$(1, 2) \rightarrow (1 - 2, 2 - 1) = (-1, 1)$

$(2, 3) \rightarrow (2 - 2, 3 - 1) = (0, 2)$

74. Answers will vary.

(a) is probably simpler to graph by plotting points and (b) is probably simpler to graph by translating the graph of $y = x^2$.

75. $\dfrac{4}{x} + \dfrac{4}{1 - x} = \dfrac{4(1 - x) + 4x}{x(1 - x)} = \dfrac{4 - 4x + 4x}{x(1 - x)} = \dfrac{4}{x(1 - x)}$

76. $\dfrac{2}{x + 5} - \dfrac{2}{x - 5}$

$$\frac{2}{x + 5} - \frac{2}{x - 5} = \frac{2(x - 5) - 2(x + 5)}{(x + 5)(x - 5)}$$

$$= \frac{2x - 10 - 2x - 10}{(x + 5)(x - 5)} = \frac{-20}{(x + 5)(x - 5)}$$

77. $\dfrac{3}{x - 1} - \dfrac{2}{x(x - 1)} = \dfrac{3x - 2}{x(x - 1)}$

78. $\dfrac{x}{x - 5} + \dfrac{1}{2}$

$$\frac{x}{x - 5} + \frac{1}{2} = \frac{2x + x - 5}{2(x - 5)} = \frac{3x - 5}{2(x - 5)}$$

79. $(x - 4)\left(\dfrac{1}{\sqrt{x^2 - 4}}\right) = \dfrac{x - 4}{\sqrt{x^2 - 4}} = \dfrac{(x - 4)\sqrt{x^2 - 4}}{x^2 - 4}$

80. $\left(\dfrac{x}{x^2 - 4}\right)\left(\dfrac{x^2 - x - 2}{x^2}\right)$

$$\left(\frac{x}{x^2 - 4}\right)\left(\frac{x^2 - x - 2}{x^2}\right) = \frac{x(x - 2)(x + 1)}{x^2(x - 2)(x + 2)}$$

$$= \frac{x + 1}{x(x + 2)}, \quad x \neq 2$$

81. $(x^2 - 9) \div \left(\dfrac{x+3}{5}\right) = \dfrac{(x+3)(x-3)}{1} \cdot \dfrac{5}{x+3} = 5(x-3),\ x \neq -3$

82. $\left(\dfrac{x}{x^2 - 3x - 28}\right) \div \left(\dfrac{x^2 + 3x}{x^2 + 5x + 4}\right)$

$$\left(\frac{x}{x^2 - 3x - 28}\right) \div \left(\frac{x^2 + 3x}{x^2 + 5x + 4}\right) = \left(\frac{x}{x^2 - 3x - 28}\right) \cdot \left(\frac{x^2 + 5x + 4}{x^2 + 3x}\right)$$

$$= \frac{x(x+4)(x+1)}{(x-7)(x+4)x(x+3)} = \frac{x+1}{(x-7)(x+3)},\quad x \neq -4, -1, 0$$

83. $f(x) = x^2 - 6x + 11$

(a) $f(-3) = (-3)^2 - 6(-3) + 11 = 38$

(b) $f\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^2 - 6\left(-\frac{1}{2}\right) + 11 = \frac{1}{4} + 3 + 11 = \frac{57}{4}$

(c) $f(x-3) = (x-3)^2 - 6(x-3) + 11 = x^2 - 6x + 9 - 6x + 18 + 11 = x^2 - 12x + 38$

84. $f(x) = \sqrt{x + 10} - 3$

(a) $f(-10) = \sqrt{-10 + 10} - 3$
$= -3$

(b) $f(26) = \sqrt{26 + 10} - 3$
$= \sqrt{36} - 3 = 3$

(c) $f(x - 10) = \sqrt{x - 10 + 10} - 3$
$= \sqrt{x} - 3$

85. $f(x) = \dfrac{2}{11 - x}$

Domain: All real numbers except $x = 11$

86. $f(x) = \dfrac{\sqrt{x-3}}{x-8}$

Domain: $x \geq 3,\ \ x \neq 8$ or $[3, 8) \cup (8, \infty)$

87. $f(x) = \sqrt{81 - x^2}$

$81 - x^2 \geq 0$

$(9 + x)(9 - x) \geq 0$

Critical numbers: $x = \pm 9$

Test intervals: $(-\infty, -9), (-9, 9), (9, \infty)$

Test: Is $81 - x^2 \geq 0$?

Solution: $[-9, 9]$

Domain of $f(x)$: $-9 \leq x \leq 9$

88. $f(x) = \sqrt[3]{4 - x^2}$

Domain: All real numbers

Section 1.8 Combinations of Functions: Composite Functions

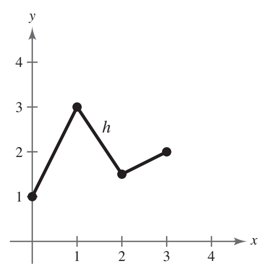
- Given two functions, f and g, you should be able to form the following functions (if defined):
  1. Sum: $(f + g)(x) = f(x) + g(x)$
  2. Difference: $(f - g)(x) = f(x) - g(x)$
  3. Product: $(fg)(x) = f(x)g(x)$
  4. Quotient: $(f/g)(x) = f(x)/g(x),\ g(x) \neq 0$
  5. Composition of f with g: $(f \circ g)(x) = f(g(x))$
  6. Composition of g with f: $(g \circ f)(x) = g(f(x))$

Vocabulary Check

1. addition, subtraction, multiplication, division
2. composition
3. $g(x)$
4. inner; outer

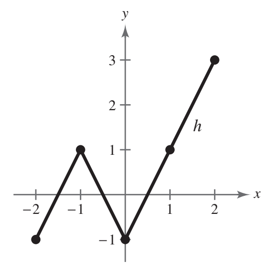
1.

x	0	1	2	3
f	2	3	1	2
g	-1	0	$\frac{1}{2}$	0
$f+g$	1	3	$\frac{3}{2}$	2



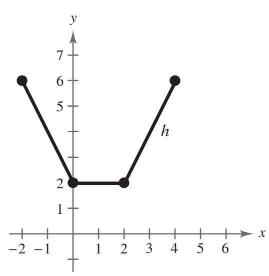
2.

x	-2	-1	0	1	2
$f(x)$	-2	0	-1	-1	1
$g(x)$	1	1	0	2	2
$h(x) = (f+g)(x)$	-1	1	-1	1	3



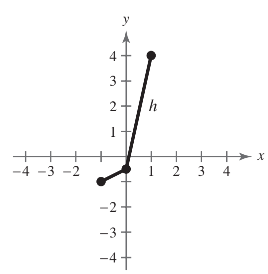
3.

x	-2	0	1	2	4
f	2	0	1	2	4
g	4	2	1	0	2
$f+g$	6	2	2	2	6



4. The domain common to both functions is $[-1, 1]$, which is the domain of the sum.

x	-1	0	1
$f(x)$	0	1.5	3
$g(x)$	-1	-2	1
$h(x) = f(x) + g(x)$	-1	-0.5	4



5. $f(x) = x + 2,\ g(x) = x - 2$

(a) $(f+g)(x) = f(x) + g(x) = (x+2) + (x-2) = 2x$

(b) $(f-g)(x) = f(x) - g(x) = (x+2) - (x-2) = 4$

(c) $(fg)(x) = f(x) \cdot g(x) = (x+2)(x-2) = x^2 - 4$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x+2}{x-2}$

Domain: all real numbers x except $x = 2$

6. $f(x) = 2x - 5,\ g(x) = 2 - x$

(a) $(f+g)(x) = 2x - 5 + 2 - x = x - 3$

(b) $(f-g)(x) = 2x - 5 - (2 - x)$

$= 2x - 5 - 2 + x = 3x - 7$

(c) $(fg)(x) = (2x - 5)(2 - x)$

$= 4x - 2x^2 - 10 + 5x = -2x^2 + 9x - 10$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{2x-5}{2-x}$

Domain: all real numbers x except $x = 2$

7. $f(x) = x^2, g(x) = 4x - 5$

(a) $(f + g)(x) = f(x) + g(x)$

$= x^2 + (4x - 5) = x^2 + 4x - 5$

(b) $(f - g)(x) = f(x) - g(x)$

$= x^2 - (4x - 5) = x^2 - 4x + 5$

(c) $(fg)(x) = f(x) \cdot g(x) = x^2(4x - 5) = 4x^3 - 5x^2$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x^2}{4x - 5}$

Domain: all real numbers x except $x = \dfrac{5}{4}$

8. $f(x) = 2x - 5, g(x) = 4$

(a) $(f + g)(x) = 2x - 5 + 4 = 2x - 1$

(b) $(f - g)(x) = 2x - 5 - 4 = 2x - 9$

(c) $(fg)(x) = (2x - 5)(4) = 8x - 20$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{2x - 5}{4} = \dfrac{1}{2}x - \dfrac{5}{4}$

Domain: all real numbers x

9. $f(x) = x^2 + 6, g(x) = \sqrt{1 - x}$

(a) $(f + g)(x) = f(x) + g(x) = (x^2 + 6) + \sqrt{1 - x}$

(b) $(f - g)(x) = f(x) - g(x) = (x^2 + 6) - \sqrt{1 - x}$

(c) $(fg)(x) = f(x) \cdot g(x) = (x^2 + 6)\sqrt{1 - x}$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x^2 + 6}{\sqrt{1 - x}} = \dfrac{(x^2 + 6)\sqrt{1 - x}}{1 - x},$

Domain: $x < 1$

10. $f(x) = \sqrt{x^2 - 4},\ g(x) = \dfrac{x^2}{x^2 + 1}$

(a) $(f + g)(x) = \sqrt{x^2 - 4} + \dfrac{x^2}{x^2 + 1}$

(b) $(f - g)(x) = \sqrt{x^2 - 4} - \dfrac{x^2}{x^2 + 1}$

(c) $(fg)(x) = \sqrt{x^2 - 4}\left(\dfrac{x^2}{x^2 + 1}\right) = \dfrac{x^2\sqrt{x^2 - 4}}{x^2 + 1}$

(d) $\left(\dfrac{f}{g}\right)(x) = \sqrt{x^2 - 4} \div \dfrac{x^2}{x^2 + 1} = \dfrac{(x^2 + 1)\sqrt{x^2 - 4}}{x^2}$

Domain: $x^2 - 4 \geq 0$

$x^2 \geq 4 \implies x \geq 2$ or $x \leq -2$

Domain: $|x| \geq 2$

11. $f(x) = \dfrac{1}{x}, g(x) = \dfrac{1}{x^2}$

(a) $(f + g)(x) = f(x) + g(x) = \dfrac{1}{x} + \dfrac{1}{x^2} = \dfrac{x + 1}{x^2}$

(b) $(f - g)(x) = f(x) - g(x) = \dfrac{1}{x} - \dfrac{1}{x^2} = \dfrac{x - 1}{x^2}$

(c) $(fg)(x) = f(x) \cdot g(x) = \dfrac{1}{x}\left(\dfrac{1}{x^2}\right) = \dfrac{1}{x^3}$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{1/x}{1/x^2} = \dfrac{x^2}{x} = x$

Domain: all real numbers x except $x = 0$

12. $f(x) = \dfrac{x}{x + 1},\ g(x) = x^3$

(a) $(f + g)(x) = \dfrac{x}{x + 1} + x^3 = \dfrac{x + x^4 + x^3}{x + 1}$

(b) $(f - g)(x) = \dfrac{x}{x + 1} - x^3 = \dfrac{x - x^4 - x^3}{x + 1}$

(c) $(fg)(x) = \dfrac{x}{x + 1} \cdot x^3 = \dfrac{x^4}{x + 1}$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{x}{x + 1} \div x^3 = \dfrac{x}{x + 1} \cdot \dfrac{1}{x^3} = \dfrac{1}{x^2(x + 1)}$

Domain: all real numbers x except $x = 0$ and $x = -1$

For Exercises 13–24, $f(x) = x^2 + 1$ and $g(x) = x - 4$.

13. $(f + g)(2) = f(2) + g(2) = (2^2 + 1) + (2 - 4) = 3$

14. $(f - g)(-1) = f(-1) - g(-1)$

$= (-1)^2 + 1 - (-1 - 4)$

$= 1 + 1 - (-5)$

$= 7$

15. $(f - g)(0) = f(0) - g(0) = (0^2 + 1) - (0 - 4) = 5$

16. $(f + g)(1) = f(1) + g(1)$
$$= (1)^2 + 1 + (1) - 4$$
$$= -1$$

17. $(f - g)(3t) = f(3t) - g(3t) = [(3t)^2 + 1] - (3t - 4)$
$$= 9t^2 - 3t + 5$$

18. $(f + g)(t - 2) = f(t - 2) + g(t - 2)$
$$= (t - 2)^2 + 1 + (t - 2) - 4$$
$$= t^2 - 4t + 4 + 1 + t - 2 - 4$$
$$= t^2 - 3t - 1$$

19. $(fg)(6) = f(6)g(6) = (6^2 + 1)(6 - 4) = 74$

20. $(fg)(-6) = f(-6) \cdot g(-6)$
$$= [(-6)^2 + 1][(-6) - 4]$$
$$= (37)(-10)$$
$$= -370$$

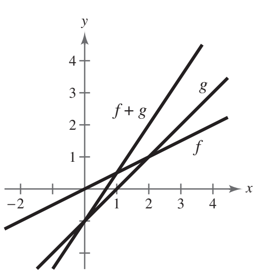
21. $\left(\frac{f}{g}\right)(5) = \frac{f(5)}{g(5)} = \frac{5^2 + 1}{5 - 4} = 26$

22. $\left(\frac{f}{g}\right)(0) = \frac{f(0)}{g(0)} = \frac{0^2 + 1}{0 - 4} = -\frac{1}{4}$

23. $\left(\frac{f}{g}\right)(-1) - g(3) = \frac{f(-1)}{g(-1)} - g(3)$
$$= \frac{(-1)^2 + 1}{-1 - 4} - (3 - 4)$$
$$= -\frac{2}{5} + 1 = \frac{3}{5}$$

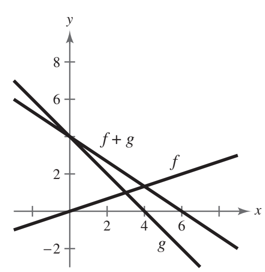
24. $(fg)(5) + f(4) = f(5)g(5) + f(4)$
$$= (5^2 + 1)(5 - 4) + (4^2 + 1)$$
$$= 26 \cdot 1 + 17$$
$$= 43$$

25. $f(x) = \frac{1}{2}x,\ g(x) = x - 1,\ (f + g)(x) = \frac{3}{2}x - 1$

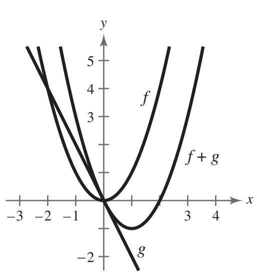


26. $f(x) = \frac{1}{3}x,\ \ g(x) = -x + 4$

$(f + g)(x) = \frac{1}{3}x - x + 4 = -\frac{2}{3}x + 4$

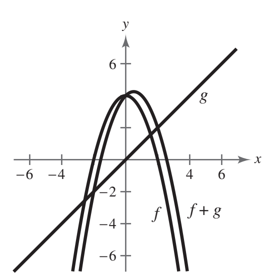


27. $f(x) = x^2,\ g(x) = -2x,\ (f + g)(x) = x^2 - 2x$

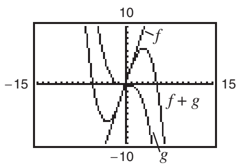


28. $f(x) = 4 - x^2,\ \ g(x) = x$

$(f + g)(x) = 4 - x^2 + x = 4 + x - x^2$



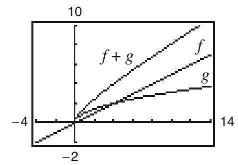
29. $f(x) = 3x, g(x) = -\dfrac{x^3}{10}, (f + g)(x) = 3x - \dfrac{x^3}{10}$



For $0 \le x \le 2$, $f(x)$ contributes most to the magnitude.

For $x > 6$, $g(x)$ contributes most to the magnitude.

30. $f(x) = \dfrac{x}{2}, \; g(x) = \sqrt{x}, (f + g)(x) = \dfrac{x}{2} + \sqrt{x}$



$g(x)$ contributes most to the magnitude of the sum for $0 \le x \le 2$. $f(x)$ contributes most to the magnitude of the sum for $x > 6$.

31. $f(x) = x^2, g(x) = x - 1$

(a) $(f \circ g)(x) = f(g(x)) = f(x - 1) = (x - 1)^2$

(b) $(g \circ f)(x) = g(f(x)) = g(x^2) = x^2 - 1$

(c) $(f \circ f)(x) = f(f(x)) = f(x^2) = (x^2)^2 = x^4$

32. $f(x) = 3x + 5, g(x) = 5 - x$

(a) $(f \circ g)(x) = f(g(x))$

$= f(5 - x) = 3(5 - x) + 5 = 20 - 3x$

(b) $(g \circ f)(x) = g(f(x))$

$= g(3x + 5) = 5 - (3x + 5) = -3x$

(c) $(f \circ f)(x) = f(f(x))$

$= f(3x + 5) = 3(3x + 5) + 5 = 9x + 20$

33. $f(x) = \sqrt[3]{x - 1}, \; g(x) = x^3 + 1$

(a) $(f \circ g)(x) = f(g(x))$

$= f(x^3 + 1)$

$= \sqrt[3]{(x^3 + 1) - 1}$

$= \sqrt[3]{x^3} = x$

(b) $(g \circ f)(x) = g(f(x))$

$= g\left(\sqrt[3]{x - 1}\right)$

$= \left(\sqrt[3]{x - 1}\right)^3 + 1$

$= (x - 1) + 1 = x$

(c) $(f \circ f)(x) = f(f(x))$

$= f\left(\sqrt[3]{x - 1}\right)$

$= \sqrt[3]{\sqrt[3]{x - 1} - 1}$

34. $f(x) = x^3, \; g(x) = \dfrac{1}{x}$

(a) $(f \circ g)(x) = f(g(x)) = f\left(\dfrac{1}{x}\right) = \left(\dfrac{1}{x}\right)^3 = \dfrac{1}{x^3}$

(b) $(g \circ f)(x) = g(f(x)) = g(x^3) = \dfrac{1}{x^3}$

(c) $(f \circ f)(x) = f(f(x)) = f(x^3) = (x^3)^3 = x^9$

35. $f(x) = \sqrt{x + 4}$ Domain: $x \ge -4$

$g(x) = x^2$ Domain: all real numbers x

(a) $(f \circ g)(x) = f(g(x)) = f(x^2) = \sqrt{x^2 + 4}$

Domain: all real numbers x

(b) $(g \circ f)(x) = g(f(x))$

$= g\left(\sqrt{x + 4}\right) = \left(\sqrt{x + 4}\right)^2 = x + 4$

Domain: $x \ge -4$

36. $f(x) = \sqrt[3]{x - 5}$ Domain: all real numbers x

$g(x) = x^3 + 1$ all real numbers x

(a) $(f \circ g)(x) = f(g(x))$

$= f(x^3 + 1) = \sqrt[3]{x^3 + 1 - 5} = \sqrt[3]{x^3 - 4}$

Domain: all real numbers x

(b) $(g \circ f)(x) = g(f(x))$

$= g\left(\sqrt[3]{x - 5}\right)$

$= \left(\sqrt[3]{x - 5}\right)^3 + 1$

$= x - 5 + 1 = x - 4$

Domain: all real numbers x

37. $f(x) = x^2 + 1$ Domain: all real numbers x

$g(x) = \sqrt{x}$ Domain: $x \geq 0$

(a) $(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 + 1 = x + 1$

Domain: $x \geq 0$

(b) $(g \circ f)(x) = g(f(x)) = g(x^2 + 1) = \sqrt{x^2 + 1}$

Domain: all real numbers x

38. $f(x) = x^{2/3}$ Domain: all real numbers x

$g(x) = x^6$ Domain: all real numbers x

(a) $(f \circ g)(x) = f(g(x)) = f(x^6) = (x^6)^{2/3} = x^4$

Domain: all real numbers x

(b) $(g \circ f)(x) = g(f(x)) = g(x^{2/3}) = (x^{2/3})^6 = x^4$

Domain: all real numbers x

39. $f(x) = |x|$ Domain: all real numbers x

$g(x) = x + 6$ Domain: all real numbers x

(a) $(f \circ g)(x) = f(g(x)) = f(x + 6) = |x + 6|$

Domain: all real numbers x

(b) $(g \circ f)(x) = g(f(x)) = g(|x|) = |x| + 6$

Domain: all real numbers x

40. $f(x) = |x - 4|$ Domain: all real numbers x

$g(x) = 3 - x$ Domain: all real numbers x

(a) $(f \circ g)(x) = f(g(x))$

$= f(3 - x) = |(3 - x) - 4| = |-x - 1|$

Domain: all real numbers x

(b) $(g \circ f)(x) = g(f(x))$

$= g(|x - 4|) = 3 - (|x - 4|) = 3 - |x - 4|$

Domain: all real numbers x

41. $f(x) = \dfrac{1}{x}$ Domain: all real numbers x except $x = 0$

$g(x) = x + 3$ Domain: all real numbers x

(a) $(f \circ g)(x) = f(g(x)) = f(x + 3) = \dfrac{1}{x + 3}$

Domain: all real numbers x except $x = -3$

(b) $(g \circ f)(x) = g(f(x)) = g\left(\dfrac{1}{x}\right) = \dfrac{1}{x} + 3$

Domain: all real numbers x except $x = 0$

42. $f(x) = \dfrac{3}{x^2 - 1}$ Domain: all real numbers x except $x = \pm 1$

$g(x) = x + 1$ Domain: all real numbers x

(a) $(f \circ g)(x) = f(g(x))$

$$= f(x + 1)$$

$$= \frac{3}{(x + 1)^2 - 1}$$

$$= \frac{3}{x^2 + 2x + 1 - 1}$$

$$= \frac{3}{x^2 + 2x}$$

Domain: all real numbers x except $x = 0$ and $x = -2$

(b) $(g \circ f)(x) = g(f(x))$

$$= g\left(\frac{3}{x^2 - 1}\right)$$

$$= \frac{3}{x^2 - 1} + 1$$

$$= \frac{3 + x^2 - 1}{x^2 - 1}$$

$$= \frac{x^2 + 2}{x^2 - 1}$$

Domain: all real numbers x except $x = \pm 1$

43. (a) $(f + g)(3) = f(3) + g(3) = 2 + 1 = 3$

(b) $\left(\dfrac{f}{g}\right)(2) = \dfrac{f(2)}{g(2)} = \dfrac{0}{2} = 0$

44. (a) $(f - g)(1) = f(1) - g(1) = 2 - 3 = -1$

(b) $(fg)(4) = f(4) \cdot g(4) = 4 \cdot 0 = 0$

45. (a) $(f \circ g)(2) = f(g(2)) = f(2) = 0$

(b) $(g \circ f)(2) = g(f(2)) = g(0) = 4$

46. (a) $(f \circ g)(1) = f(g(1)) = f(3) = 2$

(b) $(g \circ f)(3) = g(f(3)) = g(2) = 2$

47. $h(x) = (2x^2 + 1)^2$

One possibility: Let $f(x) = x^2$ and $g(x) = 2x + 1$, then $(f \circ g)(x) = h(x)$.

48. $h(x) = (1 - x)^3$

One possibility: Let $g(x) = 1 - x$ and $f(x) = x^3$, then $(f \circ g)(x) = h(x)$.

49. $h(x) = \sqrt[3]{x^2 - 4}$

One possibility: Let $f(x) = \sqrt[3]{x}$ and $g(x) = x^2 - 4$, then $(f \circ g)(x) = h(x)$.

50. $h(x) = \sqrt{9 - x}$

One possibility: Let $g(x) = 9 - x$ and $f(x) = \sqrt{x}$, then $(f \circ g)(x) = h(x)$.

51. $h(x) = \dfrac{1}{x + 2}$

One possibility: Let $f(x) = 1/x$ and $g(x) = x + 2$, then $(f \circ g)(x) = h(x)$.

52. $h(x) = \dfrac{4}{(5x + 2)^2}$

One possibility: Let $g(x) = 5x + 2$ and $f(x) = \dfrac{4}{x^2}$, then $(f \circ g)(x) = h(x)$.

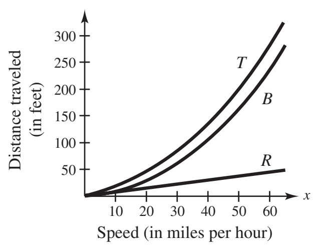
53. $h(x) = \dfrac{-x^2 + 3}{4 - x^2}$

One possibility: Let $f(x) = \dfrac{x + 3}{4 + x}$ and $g(x) = -x^2$, then $(f \circ g)(x) = h(x)$.

54. $h(x) = \dfrac{27x^3 + 6x}{10 - 27x^3}$

One possibility: Let $g(x) = x^3$ and $f(x) = \dfrac{27x + 6\sqrt[3]{x}}{10 - 27x}$, then $(f \circ g)(x) = h(x)$.

55. $T(x) = R(x) + B(x) = \frac{3}{4}x + \frac{1}{15}x^2$

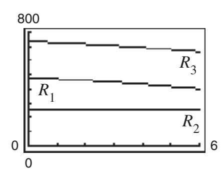


56. (a) Total sales $= R_1 + R_2$

$= 480 - 8t - 0.8t^2 + 254 + 0.78t$

$= 734 - 7.22t - 0.8t^2$

(b)



57. (a) $c(t) = \dfrac{p(t) + b(t) - d(t)}{p(t)} \times 100$

(b) $c(5)$ represents the percent change in the population in the year 2005.

58. (a) $p(t) = d(t) + c(t)$

(b) $p(5)$ represents the number of dogs and cats in 2005.

(c) $h(t) = \dfrac{p(t)}{n(t)} = \dfrac{d(t) + c(t)}{n(t)}$

$h(t)$ represents the number of dogs and cats at time t compared to the population at time t or the number of dogs and cats per capita.

59. $A(t) = 3.36t^2 - 59.8t + 735$, $N(t) = 1.95t^2 - 42.2t + 603$

(a) $(A + N)(t) = A(t) + N(t) = 5.31t^2 - 102.0t + 1338$

This represents the combined Army and Navy personnel (in thousands) from 1990 to 2002, where $t = 0$ corresponds to 1990.

$(A + N)(4) = 1014.96$ thousand

$(A + N)(8) = 861.84$ thousand

$(A + N)(12) = 878.64$ thousand

(b) $(A - N)(t) = A(t) - N(t) = 1.41t^2 - 17.6t + 132$

This represents the number of Army personnel (in thousands) more than the number of Navy personnel from 1990 to 2002, where $t = 0$ corresponds to 1990.

$(A - N)(4) = 84.16$ thousand

$(A - N)(8) = 81.44$ thousand

$(A - N)(12) = 123.84$ thousand

60. (a) $h(t) = \dfrac{E(t)}{P(t)} = \dfrac{25.95t^2 - 231.2t + 3356}{3.02t + 252.0}$

$h(t)$ represents the millions of dollars spent on exercise equipment compared to the millions of people in the U.S., or the amount spent per capita.

(b) $h(7) = 11.0169$ dollars spent per person in 1997

$h(10) = 12.895$ dollars spent per person in 2000

$h(12) = 14.982$ dollars spent per person in 2002

61.

Year	y_1	y_2	y_3
1995	146.2	329.1	44.8
1996	152.0	344.1	48.1
1997	162.2	359.9	52.1
1998	175.2	382.0	55.6
1999	184.4	412.1	57.8
2000	194.7	449.0	57.4
2001	205.5	496.1	57.8

(a) $y_1 \approx 10.20t + 92.7$

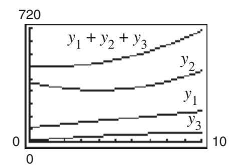
$y_2 \approx 3.357t^2 - 26.46t + 379.5$

$y_3 \approx -0.465t^2 + 9.71t + 7.4$

(b) $y_1 + y_2 + y_3 \approx 2.892t^2 - 6.55t + 479.6$

This sum represents the total spent on health services and supplies for the years 1995 through 2001. It includes out-of-pocket payments, insurance premiums, and other types of payments.

(c)



(d) For 2008 use $t = 18$:

$(y_1 + y_2 + y_3)(18) \approx \$1298.708$ billion

For 2010 use $t = 20$:

$(y_1 + y_2 + y_3)(20) \approx \$1505.4$ billion

62. (a) T is a function of t since for each time t there corresponds one and only one temperature T.

(b) $T(4) \approx 60°$; $T(15) \approx 72°$

(c) $H(t) = T(t - 1)$; All the temperature changes would be one hour later.

(d) $H(t) = T(t) - 1$; The temperature would be decreased by one degree.

(e) The points at the endpoints of the individual functions that form each "piece" appear to be $(0, 60)$, $(6, 60)$, $(7, 72)$, $(20, 72)$, $(21, 60)$, and $(24, 60)$. Note that the value $t = 24$ is chosen for the last ordered pair because that is when the day ends and the cycle starts over.

From $t = 0$ to $t = 6$: This is the constant function $T(t) = 60$.

From $t = 6$ to $t = 7$: Use the points $(6, 60)$ and $(7, 72)$.

$$m = \frac{72 - 60}{7 - 6} = 12$$

$$y - 60 = 12(x - 6) \implies y = 12x - 12, \text{ or } T(t) = 12t - 12$$

From $t = 7$ to $t = 20$: This is the constant function $T(t) = 72$.

From $t = 20$ to $t = 21$: Use the points $(20, 72)$ and $(21, 60)$.

$$m = \frac{72 - 60}{20 - 21} = -12$$

$$y - 60 = -12(x - 21) \implies y = -12x + 312, \text{ or } T(t) = -12t + 312$$

From $t = 21$ to $t = 24$: This is the constant function $T(t) = 60$.

A piecewise-defined function is $T(t) = \begin{cases} 60, & 0 \le t \le 6 \\ 12t - 12, & 6 < t < 7 \\ 72, & 7 \le t \le 20 \\ -12t + 312, & 20 < t < 21 \\ 60, & 21 \le t \le 24 \end{cases}$.

Note that the endpoints of each domain interval can be ascribed to the function on either side of it.

63. (a) $r(x) = \dfrac{x}{2}$

(b) $A(r) = \pi r^2$

(c) $(A \circ r)(x) = A(r(x)) = A\left(\dfrac{x}{2}\right) = \pi\left(\dfrac{x}{2}\right)^2$

$(A \circ r)(x)$ represents the area of the circular base of the tank on the square foundation with side length x.

64. $(A \circ r)(t) = A(r(t)) = A(0.6t) = \pi(0.6t)^2 = 0.36\pi t^2$

$A \circ r$ represents the area of the circle at time t.

65. (a) $N(T(t)) = N(3t + 2)$

$= 10(3t + 2)^2 - 20(3t + 2) + 600$

$= 10(9t^2 + 12t + 4) - 60t - 40 + 600$

$= 90t^2 + 60t + 600$

$= 30(3t^2 + 2t + 20), \quad 0 \le t \le 6$

This represents the number of bacteria in the food as a function of time.

(b) $30(3t^2 + 2t + 20) = 1500$

$3t^2 + 2t + 20 = 50$

$3t^2 + 2t - 30 = 0$

By the Quadratic Formula, $t \approx -3.513$ or 2.846. Choosing the positive value for t, we have $t \approx 2.846$ hours.

66. $C(x) = 60x + 750$, $x(t) = 50t$

(a) $(C \circ x)(t) = C(x(t))$

$= C(50t)$

$= 60(50t) + 750$

$= 3000t + 750$

$(C \circ x)(t)$ represents the cost of production as a function of time.

(b) Find t when $(C \circ x)(t) = 15{,}000$.

$15{,}000 = 3000t + 750$

$t = 4.75$ hours

The cost of production for 4 hours 45 minutes is \$15,000.

67. (a) $f(g(x)) = f(0.03x) = 0.03x - 500{,}000$

(b) $g(f(x)) = g(x - 500{,}000) = 0.03(x - 500{,}000)$

$g(f(x))$ represents your bonus of 3% of an amount over \$500,000.

68. (a) $R(p) = p - 2000$ the cost of the car after the factory rebate

(b) $S(p) = 0.9p$ the cost of the car with the dealership discount

(c) $(R \circ S)(p) = R(0.9p) = 0.9p - 2000$

$(S \circ R)(p) = S(p - 2000)$

$= 0.9(p - 2000) = 0.9p - 1800$

$(R \circ S)(p)$ represents the factory rebate *after* the dealership discount.

$(S \circ R)(p)$ represents the dealership discount after the factory rebate.

(d) $(R \circ S)(p) = (R \circ S)(20{,}500)$

$= 0.9(20{,}500) - 2000 = \$16{,}450$

$(S \circ R)(p) = (S \circ R)(20{,}500)$

$= 0.9(20{,}500) - 1800 = \$16{,}650$

$(S \circ R)(p)$ will always be larger. Observe the formulas in (c).

69. False. $(f \circ g)(x) = 6x + 1$ and $(g \circ f)(x) = 6x + 6$.

70. True. The range of g must be a subset of the domain of f for $(f \circ g)(x)$ to be defined.

71. Let $f(x)$ and $g(x)$ be two odd functions and define $h(x) = f(x)g(x)$. Then

$$\begin{aligned} h(-x) &= f(-x)g(-x) \\ &= [-f(x)][-g(x)] \quad \text{since } f \text{ and } g \text{ are odd} \\ &= f(x)g(x) \\ &= h(x). \end{aligned}$$

Thus, $h(x)$ is even.

Let $f(x)$ and $g(x)$ be two even functions and define $h(x) = f(x)g(x)$. Then

$$\begin{aligned} h(-x) &= f(-x)g(-x) \\ &= f(x)g(x) \quad \text{since } f \text{ and } g \text{ are even} \\ &= h(x). \end{aligned}$$

Thus, $h(x)$ is even.

72. Let $f(x)$ be an odd function, $g(x)$ be an even function, and define $h(x) = f(x)g(x)$. Then

$$\begin{aligned} h(-x) &= f(-x)g(-x) \\ &= [-f(x)]g(x) \quad \text{since } f \text{ is odd and } g \text{ is even} \\ &= -f(x)g(x) \\ &= -h(x). \end{aligned}$$

Thus, h is odd and the product of an odd function and an even function is odd.

73. $f(x) = 3x - 4$

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{[3(x+h) - 4] - (3x - 4)}{h} \\ &= \frac{3x + 3h - 4 - 3x + 4}{h} \\ &= \frac{3h}{h} \\ &= 3, \ h \neq 0 \end{aligned}$$

74.

$$\begin{aligned} f(x) &= 1 - x^2 \\ f(x+h) &= 1 - (x+h)^2 \\ &= 1 - (x^2 + 2hx + h^2) \\ &= 1 - x^2 - 2hx - h^2 \\ \frac{f(x+h) - f(x)}{h} &= \frac{1 - x^2 - 2hx - h^2 - (1 - x^2)}{h} \\ &= \frac{-2hx - h^2}{h} = -2x - h, h \neq 0 \end{aligned}$$

75. $f(x) = \dfrac{4}{x}$

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{\frac{4}{x+h} - \frac{4}{x}}{h} = \frac{\frac{4x - 4(x+h)}{x(x+h)}}{\frac{h}{1}} \\ &= \frac{4x - 4x - 4h}{x(x+h)} \cdot \frac{1}{h} = \frac{-4\cancel{h}}{x(x+h)} \cdot \frac{1}{\cancel{h}} = \frac{-4}{x(x+h)}, \quad h \neq 0 \end{aligned}$$

76.

$$\begin{aligned} f(x) &= \sqrt{2x+1} \\ f(x+h) &= \sqrt{2(x+h)+1} \\ \frac{f(x+h) - f(x)}{h} &= \frac{\sqrt{2(x+h)+1} - \sqrt{2x+1}}{h} \\ &= \frac{\sqrt{2(x+h)+1} - \sqrt{2x+1}}{h} \cdot \frac{\sqrt{2(x+h)+1} + \sqrt{2x+1}}{\sqrt{2(x+h)+1} + \sqrt{2x+1}} \\ &= \frac{[2(x+h)+1] - (2x+1)}{h\left(\sqrt{2(x+h)+1} + \sqrt{2x+1}\right)} \\ &= \frac{2x + 2h + 1 - 2x - 1}{h\left(\sqrt{2(x+h)+1} + \sqrt{2x+1}\right)} \\ &= \frac{2}{\sqrt{2(x+h)+1} + \sqrt{2x+1}}, h \neq 0 \end{aligned}$$

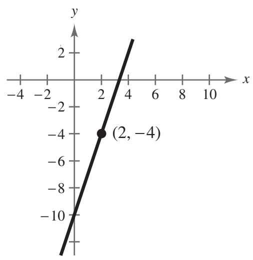
77. Point: $(2, -4)$

Slope: $m = 3$

$$y - (-4) = 3(x - 2)$$

$$y + 4 = 3x - 6$$

$$3x - y - 10 = 0$$

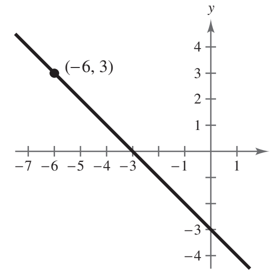


78. $(-6, 3), m = -1$

$$y - 3 = (-1)(x - (-6))$$

$$y - 3 = -x - 6$$

$$x + y + 3 = 0$$



79. Point: $(8, -1)$

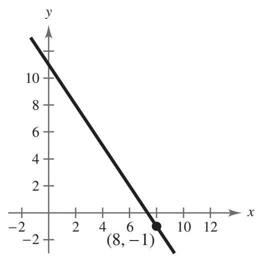
Slope: $m = -\frac{3}{2}$

$$y - (-1) = -\tfrac{3}{2}(x - 8)$$

$$y + 1 = -\tfrac{3}{2}x + 12$$

$$2y + 2 = -3x + 24$$

$$3x + 2y - 22 = 0$$

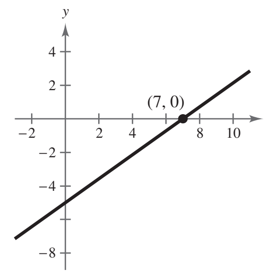


80. $(7, 0), m = \frac{5}{7}$

$$y - 0 = \tfrac{5}{7}(x - 7)$$

$$7y = 5x - 35$$

$$5x - 7y - 35 = 0$$



Section 1.9 Inverse Functions

- Two functions f and g are inverses of each other if $f(g(x)) = x$ for every x in the domain of g and $g(f(x)) = x$ for every x in the domain of f.
- A function f has an inverse function if and only if no **horizontal** line crosses the graph of f at more than one point.
- The graph of f^{-1} is a reflection of the graph of f about the line $y = x$.
- Be able to find the inverse of a function, if it exists.
  1. Use the Horizontal Line Test to see if f^{-1} exists.
  2. Replace $f(x)$ with y.
  3. Interchange x and y and solve for y.
  4. Replace y with $f^{-1}(x)$.

Vocabulary Check

1. inverse; f-inverse **2.** range; domain **3.** $y = x$

4. one-to-one **5.** Horizontal

1. $f(x) = 6x$

$$f^{-1}(x) = \frac{x}{6} = \frac{1}{6}x$$

$$f(f^{-1}(x)) = f\left(\frac{x}{6}\right) = 6\left(\frac{x}{6}\right) = x$$

$$f^{-1}(f(x)) = f^{-1}(6x) = \frac{6x}{6} = x$$

2. $f(x) = \dfrac{1}{3}x$

$$f^{-1}(x) = 3x$$

$$f(f^{-1}(x)) = f(3x) = \frac{1}{3}(3x) = x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{1}{3}x\right) = 3\left(\frac{1}{3}x\right) = x$$

3. $f(x) = x + 9$

$f^{-1}(x) = x - 9$

$f(f^{-1}(x)) = f(x - 9) = (x - 9) + 9 = x$

$f^{-1}(f(x)) = f^{-1}(x + 9) = (x + 9) - 9 = x$

4. $f(x) = x - 4$

$f^{-1}(x) = x + 4$

$f(f^{-1}(x)) = f(x + 4) = x + 4 - 4 = x$

$f^{-1}(f(x)) = f^{-1}(x - 4) = x - 4 + 4 = x$

5. $f(x) = 3x + 1$

$f^{-1}(x) = \dfrac{x - 1}{3}$

$f(f^{-1}(x)) = f\left(\dfrac{x-1}{3}\right) = 3\left(\dfrac{x-1}{3}\right) + 1 = x$

$f^{-1}(f(x)) = f^{-1}(3x + 1) = \dfrac{(3x + 1) - 1}{3} = x$

6. $f(x) = \dfrac{x - 1}{5}$

$f^{-1}(x) = 5x + 1$

$f(f^{-1}(x)) = f(5x + 1) = \dfrac{5x + 1 - 1}{5} = \dfrac{5x}{5} = x$

$f^{-1}(f(x)) = f^{-1}\left(\dfrac{x-1}{5}\right) = 5\left(\dfrac{x-1}{5}\right) + 1$

$= x - 1 + 1 = x$

7. $f(x) = \sqrt[3]{x}$

$f^{-1}(x) = x^3$

$f(f^{-1}(x)) = f(x^3) = \sqrt[3]{x^3} = x$

$f^{-1}(f(x)) = f^{-1}(\sqrt[3]{x}) = (\sqrt[3]{x})^3 = x$

8. $f(x) = x^5$

$f^{-1}(x) = \sqrt[5]{x}$

$f(f^{-1}(x)) = f(\sqrt[5]{x}) = (\sqrt[5]{x})^5 = x$

$f^{-1}(f(x)) = f^{-1}(x^5) = \sqrt[5]{x^5} = x$

9. The inverse is a line through $(-1, 0)$.
Matches graph (c).

10. The inverse is a line through $(0, 6)$ and $(6, 0)$.
Matches graph (b).

11. The inverse is half a parabola starting at $(1, 0)$.
Matches graph (a).

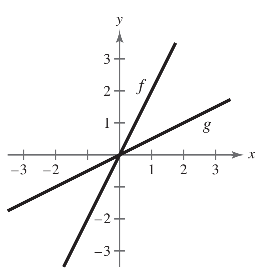
12. The inverse is a third-degree equation through $(0, 0)$.
Matches graph (d).

13. $f(x) = 2x,\ g(x) = \dfrac{x}{2}$

(a) $f(g(x)) = f\left(\dfrac{x}{2}\right) = 2\left(\dfrac{x}{2}\right) = x$

$g(f(x)) = g(2x) = \dfrac{2x}{2} = x$

(b)

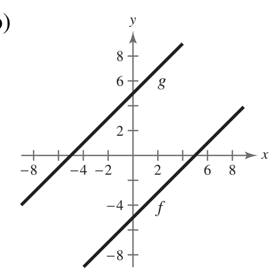


14. $f(x) = x - 5,\ g(x) = x + 5$

(a) $f(g(x)) = f(x + 5) = (x + 5) - 5 = x$

$g(f(x)) = g(x - 5) = (x - 5) + 5 = x$

(b)

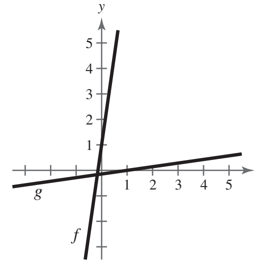


15. $f(x) = 7x + 1,\ g(x) = \dfrac{x - 1}{7}$

(a) $f(g(x)) = f\left(\dfrac{x-1}{7}\right) = 7\left(\dfrac{x-1}{7}\right) + 1 = x$

$g(f(x)) = g(7x + 1) = \dfrac{(7x + 1) - 1}{7} = x$

(b)



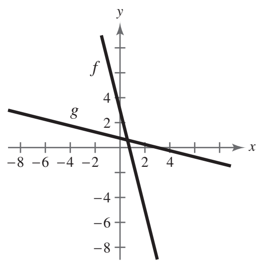
16. $f(x) = 3 - 4x,\ g(x) = \dfrac{3 - x}{4}$

(a) $f(g(x)) = f\left(\dfrac{3 - x}{4}\right) = 3 - 4\left(\dfrac{3 - x}{4}\right)$

$= 3 - (3 - x) = x$

$g(f(x)) = g(3 - 4x) = \dfrac{3 - (3 - 4x)}{4} = \dfrac{4x}{4} = x$

(b)

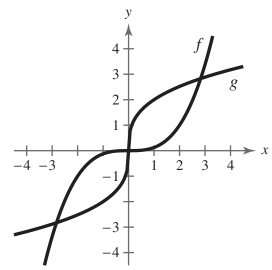


17. $f(x) = \dfrac{x^3}{8},\ g(x) = \sqrt[3]{8x}$

(a) $f(g(x)) = f\left(\sqrt[3]{8x}\right) = \dfrac{\left(\sqrt[3]{8x}\right)^3}{8} = \dfrac{8x}{8} = x$

$g(f(x)) = g\left(\dfrac{x^3}{8}\right) = \sqrt[3]{8\left(\dfrac{x^3}{8}\right)} = \sqrt[3]{x^3} = x$

(b)

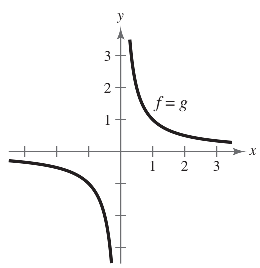


18. $f(x) = \dfrac{1}{x},\ g(x) = \dfrac{1}{x}$

(a) $f(g(x)) = f\left(\dfrac{1}{x}\right) = \dfrac{1}{1/x} = 1 \div \dfrac{1}{x} = 1 \cdot \dfrac{x}{1} = x$

$g(f(x)) = g\left(\dfrac{1}{x}\right) = \dfrac{1}{1/x} = 1 \div \dfrac{1}{x} = 1 \cdot \dfrac{x}{1} = x$

(b)



19. $f(x) = \sqrt{x - 4},\ g(x) = x^2 + 4, x \geq 0$

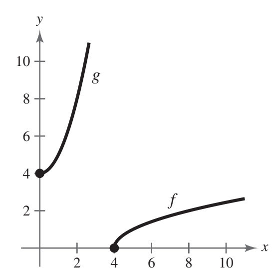
(a) $f(g(x)) = f(x^2 + 4),\ \ x \geq 0$

$= \sqrt{(x^2 + 4) - 4} = x$

$g(f(x)) = g\left(\sqrt{x - 4}\right)$

$= \left(\sqrt{x - 4}\right)^2 + 4 = x$

(b)



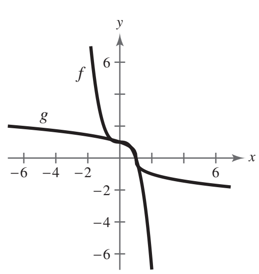
20. $f(x) = 1 - x^3,\ g(x) = \sqrt[3]{1 - x}$

(a) $f(g(x)) = f\left(\sqrt[3]{1 - x}\right) = 1 - \left(\sqrt[3]{1 - x}\right)^3$

$= 1 - (1 - x) = x$

$g(f(x)) = g(1 - x^3) = \sqrt[3]{1 - (1 - x^3)} = \sqrt[3]{x^3} = x$

(b)



21. $f(x) = 9 - x^2, x \geq 0;\ g(x) = \sqrt{9 - x}, x \leq 9$

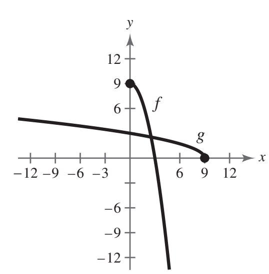
(a) $f(g(x)) = f\left(\sqrt{9 - x}\right),\ \ x \leq 9$

$= 9 - \left(\sqrt{9 - x}\right)^2 = x$

$g(f(x)) = g(9 - x^2),\ \ x \geq 0$

$= \sqrt{9 - (9 - x^2)} = x$

(b)

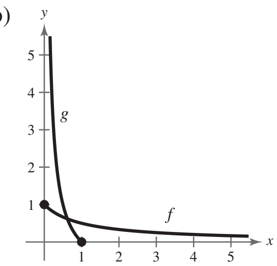


22. $f(x) = \dfrac{1}{1+x},\ x \geq 0;\ g(x) = \dfrac{1-x}{x},\quad 0 < x \leq 1$

(a) $f(g(x)) = f\left(\dfrac{1-x}{x}\right) = \dfrac{1}{1 + \left(\dfrac{1-x}{x}\right)} = \dfrac{1}{\dfrac{x}{x} + \dfrac{1-x}{x}} = \dfrac{1}{\dfrac{1}{x}} = x$

$g(f(x)) = g\left(\dfrac{1}{1+x}\right) = \dfrac{1 - \left(\dfrac{1}{1+x}\right)}{\left(\dfrac{1}{1+x}\right)} = \dfrac{\dfrac{1+x}{1+x} - \dfrac{1}{1+x}}{\dfrac{1}{1+x}} = \dfrac{\dfrac{x}{1+x}}{\dfrac{1}{1+x}} = \dfrac{x}{1+x} \cdot \dfrac{x+1}{1} = x$

(b)



23. $f(x) = \dfrac{x-1}{x+5},\ g(x) = -\dfrac{5x+1}{x-1}$

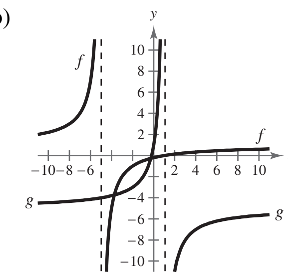
(a) $f(g(x)) = f\left(-\dfrac{5x+1}{x-1}\right)$

$= \dfrac{\left(-\dfrac{5x+1}{x-1} - 1\right)}{\left(-\dfrac{5x+1}{x-1} + 5\right)} \cdot \dfrac{x-1}{x-1} = \dfrac{-(5x+1) - (x-1)}{-(5x+1) + 5(x-1)} = \dfrac{-6x}{-6} = x$

$g(f(x)) = g\left(\dfrac{x-1}{x+5}\right)$

$= -\dfrac{\left[5\left(\dfrac{x-1}{x+5}\right) + 1\right]}{\left[\dfrac{x-1}{x+5} - 1\right]} \cdot \dfrac{x+5}{x+5} = -\dfrac{5(x-1) + (x+5)}{(x-1) - (x+5)} = -\dfrac{6x}{-6} = x$

(b)

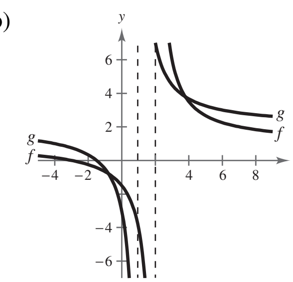


24. $f(x) = \dfrac{x+3}{x-2},\ g(x) = \dfrac{2x+3}{x-1}$

(a) $f(g(x)) = f\left(\dfrac{2x+3}{x-1}\right) = \dfrac{\dfrac{2x+3}{x-1} + 3}{\dfrac{2x+3}{x-1} - 2} = \dfrac{\dfrac{2x+3+3x-3}{x-1}}{\dfrac{2x+3-2x+2}{x-1}} = \dfrac{5x}{5} = x$

$g(f(x)) = g\left(\dfrac{x+3}{x-2}\right) = \dfrac{2\left(\dfrac{x+3}{x-2}\right) + 3}{\dfrac{x+3}{x-2} - 1} = \dfrac{\dfrac{2x+6+3x-6}{x-2}}{\dfrac{x+3-x+2}{x-2}} = \dfrac{5x}{5} = x$

(b)



25. No, $\{(-2, -1), (1, 0), (2, 1), (1, 2), (-2, 3), (-6, 4)\}$ does not represent a function. -2 and 1 are paired with two different values.

26. Yes, $\{(10, -3), (6, -2), (4, -1), (1, 0), (-3, 2), (10, 2)\}$ does represent a function.

27.

x	-2	0	2	4	6	8
$f^{-1}(x)$	-2	-1	0	1	2	3

28.

x	-10	-7	-4	-1	2	5
$f^{-1}(x)$	-3	-2	-1	0	1	2

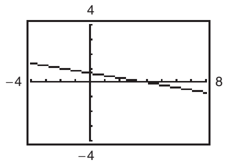
29. Yes, since no horizontal line crosses the graph of f at more than one point, f *has* an inverse.

30. No, because some horizontal lines intersect the graph twice, f *does not* have an inverse.

31. No, since some horizontal lines cross the graph of f twice, f *does not* have an inverse.

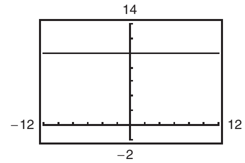
32. Yes, because no horizontal lines intersect the graph at more than one point, f *has* an inverse.

33. $g(x) = \dfrac{4 - x}{6}$



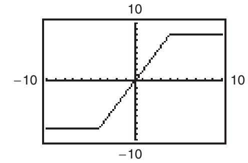
g passes the horizontal line test, so g *has* an inverse.

34. $f(x) = 10$



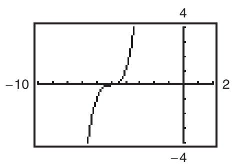
f does not pass the horizontal line test, so f *does not* have an inverse.

35. $h(x) = |x + 4| - |x - 4|$



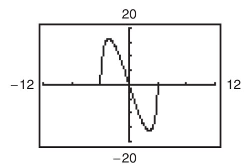
h does not pass the horizontal line test, so h *does not* have an inverse.

36. $g(x) = (x + 5)^3$



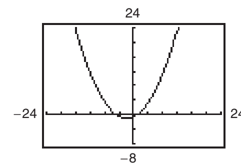
g passes the horizontal line test, so g *has* an inverse.

37. $f(x) = -2x\sqrt{16 - x^2}$



f does not pass the horizontal line test, so f *does not* have an inverse.

38. $f(x) = \frac{1}{8}(x + 2)^2 - 1$



f does not pass the horizontal line test, so f *does not* have an inverse.

39. (a) $f(x) = 2x - 3$

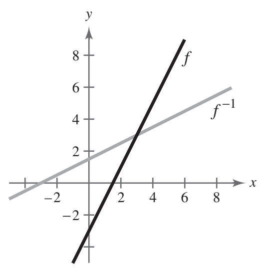
$y = 2x - 3$

$x = 2y - 3$

$y = \dfrac{x + 3}{2}$

$f^{-1}(x) = \dfrac{x + 3}{2}$

(b)



(c) The graph of f^{-1} is the reflection of the graph of f about the line $y = x$.

(d) The domains and ranges of f and f^{-1} are all real numbers.

40. (a) $f(x) = 3x + 1$

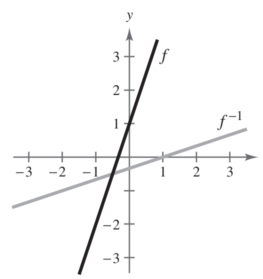
$y = 3x + 1$

$x = 3y + 1$

$\dfrac{x - 1}{3} = y$

$f^{-1}(x) = \dfrac{x - 1}{3}$

(b)



(c) The graph of f^{-1} is the reflection of f in the line $y = x$.

(d) The domains and ranges of f and f^{-1} are all real numbers.

41. (a) $f(x) = x^5 - 2$

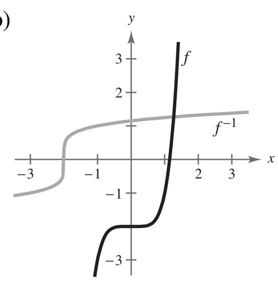
$y = x^5 - 2$

$x = y^5 - 2$

$y = \sqrt[5]{x + 2}$

$f^{-1}(x) = \sqrt[5]{x + 2}$

(b)



(c) The graph of f^{-1} is the reflection of the graph of f about the line $y = x$.

(d) The domains and ranges of f and f^{-1} are all real numbers.

42. (a) $f(x) = x^3 + 1$

$y = x^3 + 1$

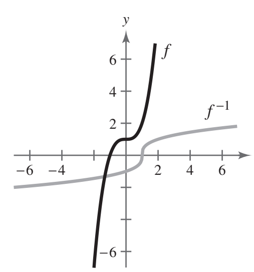
$x = y^3 + 1$

$x - 1 = y^3$

$\sqrt[3]{x - 1} = y$

$f^{-1}(x) = \sqrt[3]{x - 1}$

(b)



(c) The graph of f^{-1} is the reflection of f in the line $y = x$.

(d) The domains and ranges of f and f^{-1} are all real numbers.

43. (a) $f(x) = \sqrt{x}$

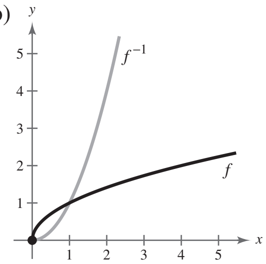
$y = \sqrt{x}$

$x = \sqrt{y}$

$y = x^2$

$f^{-1}(x) = x^2,\ x \geq 0$

(b)



(c) The graph of f^{-1} is the reflection of the graph of f about the line $y = x$.

(d) The domains and ranges of f and f^{-1} are $[0, \infty)$.

44. (a) $f(x) = x^2,\ x \geq 0$

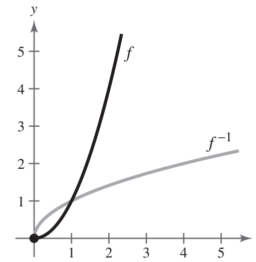
$y = x^2$

$x = y^2$

$\sqrt{x} = y$

$f^{-1}(x) = \sqrt{x}$

(b)



(c) The graph of f^{-1} is the reflection of f in the line $y = x$.

(d) The domains and ranges of f and f^{-1} are $[0, \infty)$.

45. (a) $f(x) = \sqrt{4 - x^2},\ 0 \leq x \leq 2$

$y = \sqrt{4 - x^2}$

$x = \sqrt{4 - y^2}$

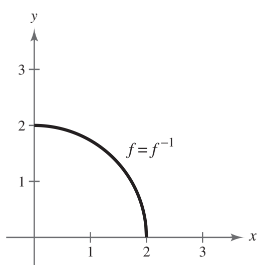
$x^2 = 4 - y^2$

$y^2 = 4 - x^2$

$y = \sqrt{4 - x^2}$

$f^{-1}(x) = \sqrt{4 - x^2},\ 0 \leq x \leq 2$

(b)



(c) The graph of f^{-1} is the same as the graph of f.

(d) The domains and ranges of f and f^{-1} are $[0, 2]$.

46. (a) $f(x) = x^2 - 2,\ \ x \leq 0$

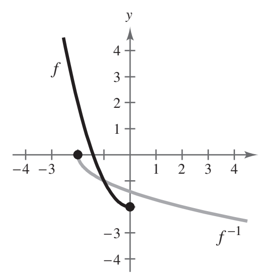
$y = x^2 - 2$

$x = y^2 - 2$

$\pm\sqrt{x + 2} = y$

$f^{-1}(x) = -\sqrt{x + 2}$

(b)



(c) The graph of f^{-1} is the reflection of f in the line $y = x$.

(d) $[-2, \infty)$ is the range of f and domain of f^{-1}. $(-\infty, 0]$ is the domain of f and the range of f^{-1}.

47. (a) $f(x) = \dfrac{4}{x}$

$y = \dfrac{4}{x}$

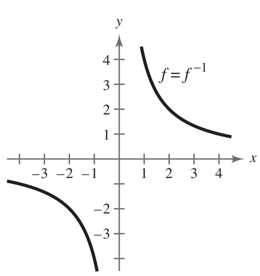
$x = \dfrac{4}{y}$

$xy = 4$

$y = \dfrac{4}{x}$

$f^{-1}(x) = \dfrac{4}{x}$

(b)



(c) The graph of f^{-1} is the same as the graph of f.

(d) The domains and ranges of f and f^{-1} are all real numbers except for 0.

48. (a) $f(x) = -\dfrac{2}{x}$

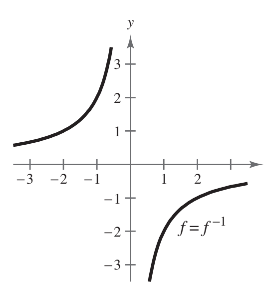
$y = -\dfrac{2}{x}$

$x = -\dfrac{2}{y}$

$y = -\dfrac{2}{x}$

$f^{-1}(x) = -\dfrac{2}{x}$

(b)



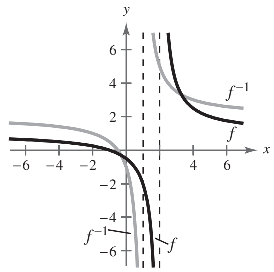
(c) The graphs are the same.

(d) $(-\infty, 0) \cup (0, \infty)$ is the domain and range of f and f^{-1}.

49. (a)

$$f(x) = \frac{x+1}{x-2}$$
$$y = \frac{x+1}{x-2}$$
$$x = \frac{y+1}{y-2}$$
$$x(y-2) = y+1$$
$$xy - 2x = y+1$$
$$xy - y = 2x+1$$
$$y(x-1) = 2x+1$$
$$y = \frac{2x+1}{x-1}$$
$$f^{-1}(x) = \frac{2x+1}{x-1}$$

(b)



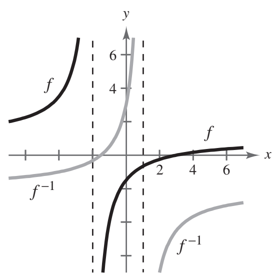
(c) The graph of f^{-1} is the reflection of the graph of f about the line $y = x$.

(d) The domain of f and the range of f^{-1} is all real numbers except 2. The range of f and the domain of f^{-1} is all real numbers except 1.

50. (a)

$$f(x) = \frac{x-3}{x+2}$$
$$y = \frac{x-3}{x+2}$$
$$x = \frac{y-3}{y+2}$$
$$xy + 2x - y + 3 = 0$$
$$y(x-1) = -2x-3$$
$$y = \frac{-2x-3}{x-1}$$
$$f^{-1}(x) = \frac{-2x-3}{x-1}$$

(b)



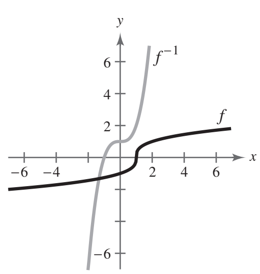
(c) The graph of f^{-1} is the reflection of the graph of f about the line $y = x$.

(d) The domain of f and the range of f^{-1} is all real numbers except $x = -2$. The range of f and the domain of f^{-1} is all real numbers except $x = 1$.

51. (a)

$$f(x) = \sqrt[3]{x-1}$$
$$y = \sqrt[3]{x-1}$$
$$x = \sqrt[3]{y-1}$$
$$x^3 = y-1$$
$$y = x^3+1$$
$$f^{-1}(x) = x^3+1$$

(b)



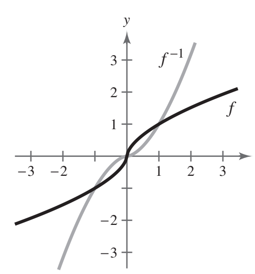
(c) The graph of f^{-1} is the reflection of the graph of f about the line $y = x$.

(d) The domains and ranges of f and f^{-1} are all real numbers.

52. (a)

$$f(x) = x^{3/5}$$
$$y = x^{3/5}$$
$$x = y^{3/5}$$
$$x^{5/3} = (y^{3/5})^{5/3}$$
$$x^{5/3} = y$$
$$f^{-1}(x) = x^{5/3}$$

(b)



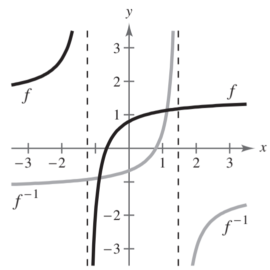
(c) The graph of f^{-1} is the reflection of the graph of f about the line $y = x$.

(d) The set of all real numbers is the domain and range of f and f^{-1}.

53. (a)

$$\begin{aligned} f(x) &= \frac{6x+4}{4x+5} \\ y &= \frac{6x+4}{4x+5} \\ x &= \frac{6y+4}{4y+5} \\ x(4y+5) &= 6y+4 \\ 4xy+5x &= 6y+4 \\ 4xy-6y &= -5x+4 \\ y(4x-6) &= -5x+4 \\ y &= \frac{-5x+4}{4x-6} \\ f^{-1}(x) &= \frac{-5x+4}{4x-6} = \frac{5x-4}{6-4x} \end{aligned}$$

(b)



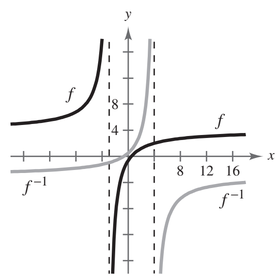
(c) The graph of f^{-1} is the graph of f reflected about the line $y = x$.

(d) The domain of f and the range of f^{-1} is all real numbers except $-\frac{5}{4}$.
The range of f and the domain of f^{-1} is all real numbers except $\frac{3}{2}$.

54. (a)

$$\begin{aligned} f(x) &= \frac{8x-4}{2x+6} \\ y &= \frac{8x-4}{2x+6} \\ x &= \frac{8y-4}{2y+6} \\ 2xy+6x &= 8y-4 \\ y(2x-8) &= -6x-4 \\ y &= \frac{-6x-4}{2x-8} \end{aligned}$$

(b)



(c) The graph of f^{-1} is the graph of f reflected about the line $y = x$.

(d) The domain of f and the range of f^{-1} is the set of all real numbers x except $x = -3$.
The domain of f^{-1} and the range of f is the set of all real numbers x except $x = 4$.

55. $f(x) = x^4$

$$\begin{aligned} y &= x^4 \\ x &= y^4 \\ y &= \pm\sqrt[4]{x} \end{aligned}$$

This does not represent y as a function of x. f does not have an inverse.

56. $f(x) = \dfrac{1}{x^2}$

$$\begin{aligned} y &= \frac{1}{x^2} \\ x &= \frac{1}{y^2} \\ y^2 &= \frac{1}{x} \\ y &= \pm\sqrt{\frac{1}{x}} \end{aligned}$$

This does not represent y as a function of x. f does not have an inverse.

57. $g(x) = \dfrac{x}{8}$

$$\begin{aligned} y &= \frac{x}{8} \\ x &= \frac{y}{8} \\ y &= 8x \end{aligned}$$

This is a function of x, so g has an inverse.

$g^{-1}(x) = 8x$

58. $f(x) = 3x + 5$

$y = 3x + 5$

$x = 3y + 5$

$x - 5 = 3y$

$\dfrac{x - 5}{3} = y$

This is a function of x, so f has an inverse.

$f^{-1}(x) = \dfrac{x - 5}{3}$

59. $p(x) = -4$

$y = -4$

Since $y = -4$ for all x, the graph is a horizontal line and fails the Horizontal Line Test. p does not have an inverse.

60. $f(x) = \dfrac{3x + 4}{5}$

$y = \dfrac{3x + 4}{5}$

$x = \dfrac{3y + 4}{5}$

$5x = 3y + 4$

$5x - 4 = 3y$

$\dfrac{5x - 4}{3} = y$

This is a function of x, so f has an inverse.

$f^{-1}(x) = \dfrac{5x - 4}{3}$

61. $f(x) = (x + 3)^2,\ x \ge -3 \implies y \ge 0$

$y = (x + 3)^2,\ x \ge -3,\ y \ge 0$

$x = (y + 3)^2,\ y \ge -3,\ x \ge 0$

$\sqrt{x} = y + 3,\ y \ge -3,\ x \ge 0$

$y = \sqrt{x} - 3,\ x \ge 0,\ y \ge -3$

This is a function of x, so f has an inverse.

$f^{-1}(x) = \sqrt{x} - 3,\ x \ge 0$

62. $q(x) = (x - 5)^2$

$y = (x - 5)^2$

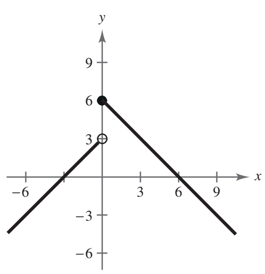
$x = (y - 5)^2$

$\pm\sqrt{x} = y - 5$

$5 \pm \sqrt{x} = y$

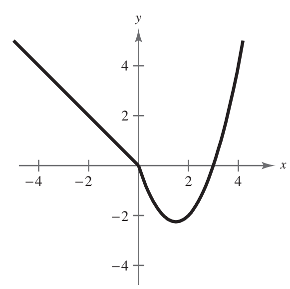
This does not represent y as a function of x, so q does not have an inverse.

63. $f(x) = \begin{cases} x + 3, & x < 0 \\ 6 - x, & x \ge 0 \end{cases}$



The graph fails the Horizontal Line Test, so $f(x)$ does not have an inverse.

64. $f(x) = \begin{cases} -x, & x \le 0 \\ x^2 - 3x, & x > 0 \end{cases}$



The graph fails the Horizontal Line Test, so f does not have an inverse.

65. $h(x) = -\dfrac{4}{x^2}$

The graph fails the Horizontal Line Test so h does not have an inverse.

66. $f(x) = |x - 2|,\ x \le 2 \implies y \ge 0$

$y = |x - 2|,\ x \le 2,\ y \ge 0$

$x = |y - 2|,\ y \le 2,\ x \ge 0$

$x = y - 2$ or $-x = y - 2$

$2 + x = y$ or $2 - x = y$

The portion that satisfies the conditions $y \le 2$ and $x \ge 0$ is $2 - x = y$. This is a function of x, so f has an inverse.

$f^{-1}(x) = 2 - x,\ x \ge 0$

67. $f(x) = \sqrt{2x+3} \implies x \geq -\dfrac{3}{2},\ y \geq 0$

$$y = \sqrt{2x+3},\ x \geq -\frac{3}{2},\ y \geq 0$$

$$x = \sqrt{2y+3},\ y \geq -\frac{3}{2},\ x \geq 0$$

$$x^2 = 2y + 3,\ x \geq 0,\ y \geq -\frac{3}{2}$$

$$y = \frac{x^2-3}{2},\ x \geq 0,\ y \geq -\frac{3}{2}$$

This is a function of x, so f has an inverse.

$$f^{-1}(x) = \frac{x^2-3}{2},\ x \geq 0$$

68. $f(x) = \sqrt{x-2} \implies x \geq 2,\ y \geq 0$

$$y = \sqrt{x-2},\ x \geq 2,\ y \geq 0$$

$$x = \sqrt{y-2},\ y \geq 2,\ x \geq 0$$

$$x^2 = y - 2,\ x \geq 0,\ y \geq 2$$

$$x^2 + 2 = y,\ x \geq 0,\ y \geq 2$$

This is a function of x, so f has an inverse.

$$f^{-1}(x) = x^2 + 2,\ x \geq 0$$

In Exercises 69–74, $f(x) = \frac{1}{8}x - 3$, $f^{-1}(x) = 8(x+3)$, $g(x) = x^3$, $g^{-1}(x) = \sqrt[3]{x}$.

69.
$$\begin{aligned}(f^{-1} \circ g^{-1})(1) &= f^{-1}(g^{-1}(1)) \\ &= f^{-1}\left(\sqrt[3]{1}\right) \\ &= 8\left(\sqrt[3]{1} + 3\right) = 32\end{aligned}$$

70.
$$\begin{aligned}(g^{-1} \circ f^{-1})(-3) &= g^{-1}(f^{-1}(-3)) \\ &= g^{-1}(8(-3+3)) \\ &= g^{-1}(0) = \sqrt[3]{0} = 0\end{aligned}$$

71.
$$\begin{aligned}(f^{-1} \circ f^{-1})(6) &= f^{-1}(f^{-1}(6)) \\ &= f^{-1}(8[6+3]) \\ &= 8[8(6+3)+3] = 600\end{aligned}$$

72.
$$\begin{aligned}(g^{-1} \circ g^{-1})(-4) &= g^{-1}(g^{-1}(-4)) \\ &= g^{-1}\left(\sqrt[3]{-4}\right) \\ &= \sqrt[3]{\sqrt[3]{-4}} = \sqrt[9]{-4}\end{aligned}$$

73.
$$\begin{aligned}(f \circ g)(x) &= f(g(x)) = f(x^3) = \tfrac{1}{8}x^3 - 3 \\ y &= \tfrac{1}{8}x^3 - 3 \\ x &= \tfrac{1}{8}y^3 - 3 \\ x + 3 &= \tfrac{1}{8}y^3 \\ 8(x+3) &= y^3 \\ \sqrt[3]{8(x+3)} &= y \\ (f \circ g)^{-1}(x) &= 2\sqrt[3]{x+3}\end{aligned}$$

74.
$$\begin{aligned}g^{-1} \circ f^{-1} &= g^{-1}(f^{-1}(x)) \\ &= g^{-1}(8(x+3)) \\ &= \sqrt[3]{8(x+3)} \\ &= 2\sqrt[3]{x+3}\end{aligned}$$

In Exercises 75–78, $f(x) = x + 4$, $f^{-1}(x) = x - 4$, $g(x) = 2x - 5$, $g^{-1}(x) = \dfrac{x+5}{2}$.

75.
$$\begin{aligned}(g^{-1} \circ f^{-1})(x) &= g^{-1}(f^{-1}(x)) \\ &= g^{-1}(x-4) \\ &= \frac{(x-4)+5}{2} \\ &= \frac{x+1}{2}\end{aligned}$$

76.
$$\begin{aligned}(f^{-1} \circ g^{-1})(x) &= f^{-1}(g^{-1}(x)) \\ &= f^{-1}\left(\frac{x+5}{2}\right) \\ &= \frac{x+5}{2} - 4 \\ &= \frac{x+5-8}{2} \\ &= \frac{x-3}{2}\end{aligned}$$

77.
$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(2x-5) \\ &= (2x-5)+4 \\ &= 2x - 1\end{aligned}$$

$$(f \circ g)^{-1}(x) = \frac{x+1}{2}$$

Note: Comparing Exercises 75 and 77, we see that $(f \circ g)^{-1}(x) = (g^{-1} \circ f^{-1})(x)$.

78. $$\begin{aligned}(g \circ f)(x) &= g(f(x))\\ &= g(x + 4)\\ &= 2(x + 4) - 5\\ &= 2x + 8 - 5\\ &= 2x + 3\end{aligned}$$

$$y = 2x + 3$$
$$x = 2y + 3$$
$$x - 3 = 2y$$
$$\frac{x-3}{2} = y$$
$$(g \circ f)^{-1}(x) = \frac{x-3}{2}$$

79. (a) $f^{-1}(108{,}209) = 11$

(b) f^{-1} represents the year for a given number of households in the United States.

(c) $y \approx 1578.68t + 90{,}183.63$

(d) $$y = 1578.68t + 90{,}183.63$$
$$t = 1578.68y + 90{,}183.63$$
$$\frac{t - 90{,}183.63}{1578.68} = y$$
$$f^{-1}(t) = \frac{t - 90{,}183.63}{1578.68}$$

(e) $f^{-1}(117{,}022) \approx 17$

(f) $f^{-1}(108{,}209) \approx 11.418$

This is close to the value of 11 in the table.

80. (a) Yes, f^{-1} exists.

(b) f^{-1} represents the time in years for a given total sales.

(c) $f^{-1}(1825) = 10$

(d) No. f^{-1} would not exist since $f(12) = 2794$ and $f(14) = 2794$. The function would fail the Horizontal Line Test.

81. (a) Yes. Since the values of f increase each year, no two f-values are paired with the same t-value so f does have an inverse.

(b) f^{-1} would represent the year that a given number of miles was traveled by motor vehicles.

(c) Since $f(8) = 2632$, $f^{-1}(2632) = 8$.

(d) No. Since the new value is the same as the value given for 2000, f would not pass the Horizontal Line Test and would not have an inverse.

82. (a) $$y = 8 + 0.75x$$
$$x = 8 + 0.75y$$
$$x - 8 = 0.75y$$
$$\frac{x-8}{0.75} = y$$
$$f^{-1}(x) = \frac{x-8}{0.75}$$

(b) $x =$ hourly wage, $y =$ number of units produced

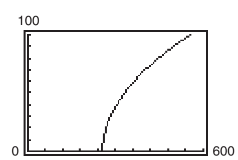
(c) $y = \dfrac{22.25 - 8}{0.75} = 19$ units

83. (a) $$y = 0.03x^2 + 245.50,\ 0 < x < 100$$
$$\implies 245.50 < y < 545.50$$
$$x = 0.03y^2 + 245.50$$
$$x - 245.50 = 0.03y^2$$
$$\frac{x - 245.50}{0.03} = y^2$$
$$\sqrt{\frac{x - 245.50}{0.03}} = y,\ 245.50 < x < 545.50$$
$$f^{-1}(x) = \sqrt{\frac{x - 245.50}{0.03}}$$

$x =$ temperature in degrees Fahrenheit

$y =$ percent load for a diesel engine

(b)



(c) $$0.03x^2 + 245.50 \le 500$$
$$0.03x^2 \le 254.50$$
$$x^2 \le 8483.33$$
$$x \le 92.10$$

Thus, $0 < x \le 92.10$.

84. (a)

$$x = 1.25y + 1.60(50 - y)$$
$$x = 1.25y + 80 - 1.60y$$
$$x - 80 = -0.35y$$
$$\frac{x - 80}{-0.35} = y$$
$$y = \frac{80 - x}{0.35}$$

$x =$ total cost

$y =$ number of pounds of less expensive ground beef

(b)

$$0 \le y \le 50$$
$$0 \le \frac{80 - x}{0.35} \le 50$$
$$0 \le 80 - x \le 17.5$$
$$-80 \le -x \le -62.5$$
$$62.5 \le x \le 80$$

(c) $\frac{80 - 73}{0.35} = y = 20$ pounds of the less expensive ground beef

85. False. $f(x) = x^2$ is even and does not have an inverse.

86. True. If $f(x)$ has an inverse and it has a y-intercept at $(0, b)$, then the point $(b, 0)$ must be a point on the graph of $f^{-1}(x)$.

87. Let $(f \circ g)(x) = y$. Then $x = (f \circ g)^{-1}(y)$. Also,

$$(f \circ g)(x) = y \implies f(g(x)) = y$$
$$g(x) = f^{-1}(y)$$
$$x = g^{-1}(f^{-1}(y))$$
$$x = (g^{-1} \circ f^{-1})(y).$$

Since f and g are both one-to-one functions,

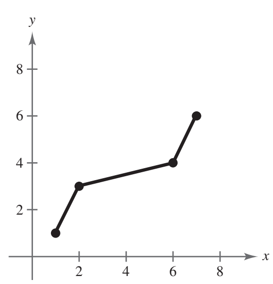
$(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

88. Let $f(x)$ be a one-to-one odd function. Then $f^{-1}(x)$ exists and $f(-x) = -f(x)$. Letting (x, y) be any point on the graph of $f(x) \implies (-x, -y)$ is also on the graph of $f(x)$ and $f^{-1}(-y) = -x = -f^{-1}(y)$. Therefore, $f^{-1}(x)$ is also an odd function.

89.

x	1	3	4	6
f	1	2	6	7

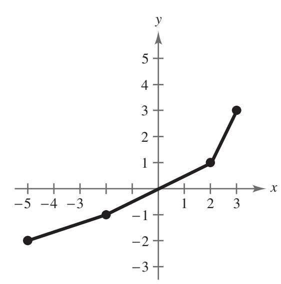
x	1	2	6	7
$f^{-1}(x)$	1	3	4	6



90.

x	-2	-1	1	3
$f(x)$	-5	-2	2	3

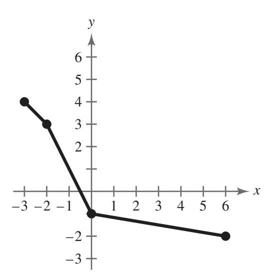
x	-5	-2	2	3
$f^{-1}(x)$	-2	-1	1	3



91.

x	-2	-1	3	4
f	6	0	-2	-3

x	-3	-2	0	6
$f^{-1}(x)$	4	3	-1	-2



92.

x	-4	-2	0	3
$f(x)$	3	4	0	-1

The graph does not pass the Horizontal Line Test, so $f^{-1}(x)$ does not exist.

93. If $f(x) = k(2 - x - x^3)$ has an inverse and $f^{-1}(3) = -2$, then $f(-2) = 3$. Thus,

$$f(-2) = k(2 - (-2) - (-2)^3) = 3$$
$$k(2 + 2 + 8) = 3$$
$$12k = 3$$
$$k = \tfrac{3}{12} = \tfrac{1}{4}$$

So, $k = \frac{1}{4}$.

94. $f(x) = k(x^3 + 3x - 4)$

$$y = k(x^3 + 3x - 4)$$
$$x = k(y^3 + 3y - 4)$$
$$-5 = k[(2)^3 + 3(2) - 4)]$$
$$-5 = 10k$$
$$-\tfrac{1}{2} = k$$

95. $x^2 = 64$

$$x = \pm\sqrt{64} = \pm 8$$

96. $(x - 5)^2 = 8$

$$x - 5 = \pm\sqrt{8}$$
$$x = 5 \pm 2\sqrt{2}$$

97. $4x^2 - 12x + 9 = 0$

$$(2x - 3)^2 = 0$$
$$2x - 3 = 0$$
$$x = \tfrac{3}{2}$$

98. $9x^2 + 12x + 3 = 0$

$$(9x + 3)(x + 1) = 0$$
$$9x + 3 = 0 \implies x = -\tfrac{1}{3}$$
$$x + 1 = 0 \implies x = -1$$

99. $x^2 - 6x + 4 = 0$ Complete the square.

$$x^2 - 6x = -4$$
$$x^2 - 6x + 9 = -4 + 9$$
$$(x - 3)^2 = 5$$
$$x - 3 = \pm\sqrt{5}$$
$$x = 3 \pm \sqrt{5}$$

100. $2x^2 - 4x - 6 = 0$

$$2(x^2 - 2x - 3) = 0$$
$$2(x + 1)(x - 3) = 0$$
$$x + 1 = 0 \implies x = -1$$
$$x - 3 = 0 \implies x = 3$$

101. $50 + 5x = 3x^2$

$$0 = 3x^2 - 5x - 50$$
$$0 = (3x + 10)(x - 5)$$
$$3x + 10 = 0 \implies x = -\tfrac{10}{3}$$
$$x - 5 = 0 \implies x = 5$$

102. $2x^2 + 4x - 9 = 2(x - 1)^2$

$$2x^2 + 4x - 9 = 2(x^2 - 2x + 1)$$
$$2x^2 + 4x - 9 = 2x^2 - 4x + 2$$
$$8x - 11 = 0$$
$$8x = 11$$
$$x = \tfrac{11}{8}$$

103. Let $2n$ = first positive even integer. Then $2n + 2$ = next positive even integer.

$$2n(2n + 2) = 288$$
$$4n^2 + 4n - 288 = 0$$
$$4(n^2 + n - 72) = 0$$
$$4(n + 9)(n - 8) = 0$$

$n + 9 = 0 \implies n = -9$ Not a solution since the integers are positive.

$n - 8 = 0 \implies n = 8$

So, $2n = 16$ and $2n + 2 = 18$.

104. Given $h = 2b$ and $A = 10$

$$A = \frac{1}{2}bh$$

$$10 = \frac{1}{2}b(2b)$$

$$10 = b^2$$

$\sqrt{10} = b$ and $h = 2b = 2\sqrt{10}$

The base is $\sqrt{10}$ feet and the height is $2\sqrt{10}$ feet.

Section 1.10 Mathematical Modeling and Variation

You should know the following the following terms and formulas.

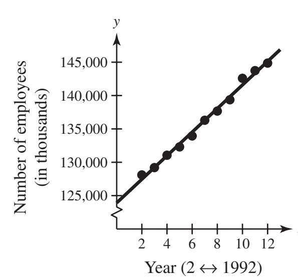
- Direct variation (varies directly, directly proportional)
  (a) $y = kx$ (b) $y = kx^n$ (as nth power)
- Inverse variation (varies inversely, inversely proportional)
  (a) $y = k/x$ (b) $y = k/(x^n)$ (as nth power)
- Joint variation (varies jointly, jointly proportional)
  (a) $z = kxy$ (b) $z = kx^ny^m$ (as nth power of x and mth power of y)
- k is called the constant of proportionality.
- Least Squares Regression Line $y = ax + b$. Use your calculator or computer to enter the data points and to find the "best-fitting"linear model.

Vocabulary Check

1. variation; regression
2. sum of square differences
3. correlation coefficient
4. directly proportional
5. constant of variation
6. directly proportional
7. inverse
8. combined
9. jointly proportional

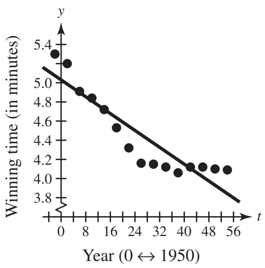
1. $y = 1767.0t + 123{,}916$

Year	Actual Number (in thousands)	Model (in thousands)
1992	128,105	127,450
1993	129,200	129,217
1994	131,056	130,984
1995	132,304	132,751
1996	133,943	134,518
1997	136,297	136,285
1998	137,673	138,052
1999	139,368	139,819
2000	142,583	141,586
2001	143,734	143,353
2002	144,863	145,120



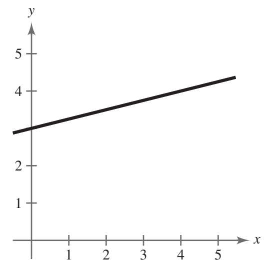
The model is a good fit for the actual data.

2.



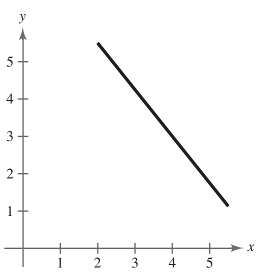
The model is not a "good fit" for the actual data. It appears that another type of model may be a better fit.

3.



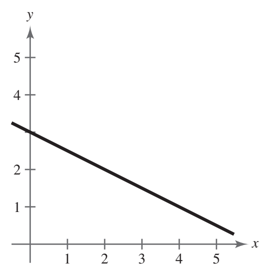
Using the points $(0, 3)$ and $(4, 4)$, we have $y = \frac{1}{4}x + 3$.

4.



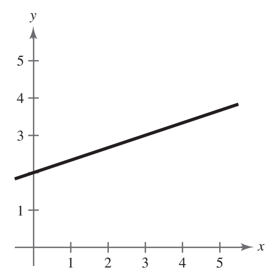
The line appears to pass through $(2, 5.5)$ and $(6, 0.5)$, so its equation is $y = -\frac{5}{4}x + 8$.

5.



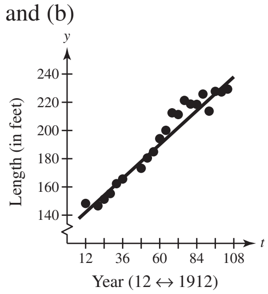
Using the points $(2, 2)$ and $(4, 1)$, we have $y = -\frac{1}{2}x + 3$.

6.



The line appears to pass through $(0, 2)$ and $(3, 3)$ so its equation is $y = \frac{1}{3}x + 2$.

7. (a) and (b)



$y \approx t + 130$

(c) $y \approx 1.03t + 130.27$

(d) The models are similar.

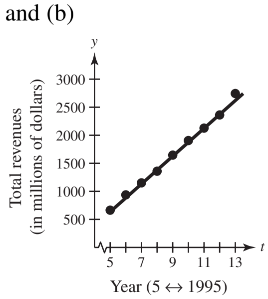
(e) When $t = 108$, we have:

Model in part (b): 238 feet

Model in part (c): 241.51 feet

(f) Answers will vary.

8. (a) and (b)



(b) The line appears to pass through $(7, 1151.6)$ and $(10, 1906.0)$, so the equation is about $y = 251.5x - 609$.

(c) $y = 251.56x - 608.79$

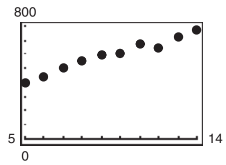
(d) Answers will vary.

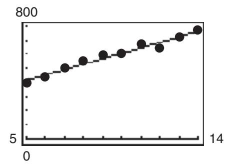
(e) Using the model in (b), $y = 251.5(15) - 609 = \$3164.6$ million.

Using the model in (c), $y = 251.56(15) - 608.79 = \$3165.2$ million.

(f) Answers will vary.

9. (a) and (c)





The model is a good fit to the actual data. ($r \approx 0.98$)

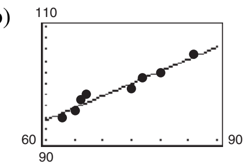
(b) $S \approx 38.4t + 224$

(d) For 2005, use $t = 15$: $S \approx \$800.4$ million
For 2007, use $t = 17$: $S \approx \$877.3$ million

(e) Each year the annual gross ticket sales for Broadway shows in New York City increase by approximately \$38.4 million.

10. (a) $y = 0.4306x + 67.708$

(b)



The model is a good fit to the data. ($r \approx 0.97$)

(c) $y = 0.4306(90) + 67.708 = 106.5$ million

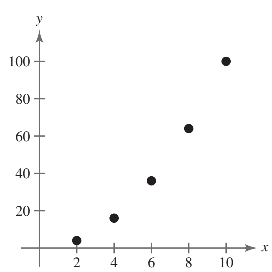
(d) For every increase of one million households with cable TV, there is a 0.43 million increase in the number of households with color TV.

11. The graph appears to represent $y = 4/x$, so y varies inversely as x.

12. The graph appears to represent $y = \frac{3}{2}x$ which is a direct variation.

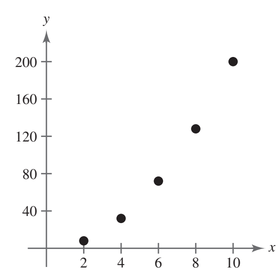
13. $k = 1$

x	2	4	6	8	10
$y = kx^2$	4	16	36	64	100



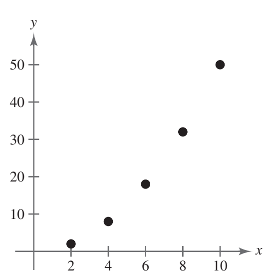
14. $k = 2$

x	2	4	6	8	10
$y = kx^2$	8	32	72	128	200



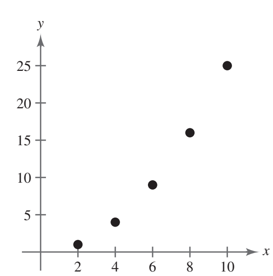
15. $k = \frac{1}{2}$

x	2	4	6	8	10
$y = kx^2$	2	8	18	32	50



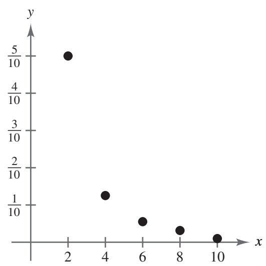
16. $k = \frac{1}{4}$

x	2	4	6	8	10
$y = kx^2$	1	4	9	16	25



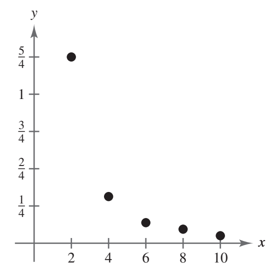
17. $k = 2$

x	2	4	6	8	10
$y = \frac{k}{x^2}$	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{18}$	$\frac{1}{32}$	$\frac{1}{50}$



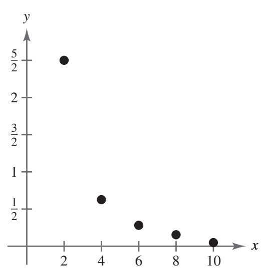
18. $k = 5$

x	2	4	6	8	10
$y = \frac{k}{x^2}$	$\frac{5}{4}$	$\frac{5}{16}$	$\frac{5}{36}$	$\frac{5}{64}$	$\frac{1}{20}$



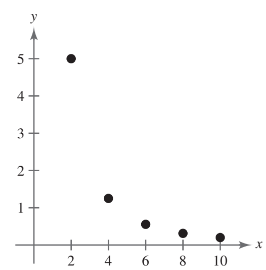
19. $k = 10$

x	2	4	6	8	10
$y = \frac{k}{x^2}$	$\frac{5}{2}$	$\frac{5}{8}$	$\frac{5}{18}$	$\frac{5}{32}$	$\frac{1}{10}$



20. $k = 20$

x	2	4	6	8	10
$y = \frac{k}{x^2}$	5	$\frac{5}{4}$	$\frac{5}{9}$	$\frac{5}{16}$	$\frac{1}{5}$



21. The table represents the equation $y = 5/x$.

22. The table represents the equation $y = \frac{2}{5}x$.

23.
$$\begin{aligned} y &= kx \\ -7 &= k(10) \\ -\frac{7}{10} &= k \\ y &= -\frac{7}{10}x \end{aligned}$$

This equation checks with the other points given in the table.

24.
$$\begin{aligned} y &= \frac{k}{x} \\ 24 &= \frac{k}{5} \\ 120 &= k \end{aligned}$$

Thus, $y = 120/x$. This equation checks with the other points given in the table.

25.
$$\begin{aligned} y &= kx \\ 12 &= k(5) \\ \frac{12}{5} &= k \\ y &= \frac{12}{5}x \end{aligned}$$

26.
$$\begin{aligned} y &= kx \\ 14 &= k(2) \\ 7 &= k \\ y &= 7x \end{aligned}$$

27.
$$\begin{aligned} y &= kx \\ 2050 &= k(10) \\ 205 &= k \\ y &= 205x \end{aligned}$$

28.
$$\begin{aligned} y &= kx \\ 580 &= k(6) \\ \tfrac{290}{3} &= k \\ y &= \tfrac{290}{3}x \end{aligned}$$

29.
$$\begin{aligned} I &= kP \\ 87.50 &= k(2500) \\ 0.035 &= k \\ I &= 0.035P \end{aligned}$$

30. $I = kP$

$187.50 = k(5000)$

$0.0375 = k$

$I = 0.0375P$

31. $y = kx$

$33 = k(13)$

$\frac{33}{13} = k$

$y = \frac{33}{13}x$

When $x = 10$ inches, $y \approx 25.4$ centimeters.

When $x = 20$ inches, $y \approx 50.8$ centimeters.

32. $y = kx$

$53 = k(14)$

$\frac{53}{14} = k$

$y = \frac{53}{14}x$

5 gallons: $y = \frac{53}{14}(5) \approx 18.9$ liters

25 gallons: $y = \frac{53}{14}(25) \approx 94.6$ liters

33. $y = kx$

$5520 = k(150{,}000)$

$0.0368 = k$

$y = 0.0368x$

$y = 0.0368(200{,}000)$

$= \$7360$

The property tax is \$7360.

34. $y = kx$

$10.22 = k(145.99)$

$0.07 \approx k$

$y = 0.07x$

$y = 0.07(540.50)$

$y \approx 37.84$

The sales tax is \$37.84.

35. $d = kF$

$0.15 = k(265)$

$\frac{3}{5300} = k$

$d = \frac{3}{5300}F$

(a) $d = \frac{3}{5300}(90) \approx 0.05$ meter

(b) $0.1 = \frac{3}{5300}F$

$\frac{530}{3} = F$

$F = 176\frac{2}{3}$ newtons

36. $d = kF$

$0.12 = k(220)$

$\frac{3}{5500} = k$

$d = \frac{3}{5500}F$

$0.16 = \frac{3}{5500}F$

$\frac{880}{3} = F$

The required force is $293\frac{1}{3}$ newtons.

37. $d = kF$

$1.9 = k(25) \implies k = 0.076$

$d = 0.076F$

When the distance compressed is 3 inches, we have

$3 = 0.076F$

$F \approx 39.47.$

No child over 39.47 pounds should use the toy.

38. $d = kF$

$1 = k(15)$

$k = \frac{1}{15}$

$d = \frac{1}{15}F$

$\frac{8}{2} = \frac{1}{15}F$

$F = 60$ lb per spring

Combined lifting force $= 2F$

$= 120$ lbs

39. $A = kr^2$

40. $V = ke^3$

41. $y = \dfrac{k}{x^2}$

42. $h = \dfrac{k}{\sqrt{s}}$

43. $F = \dfrac{kg}{r^2}$

44. $z = kx^2y^3$

45. $P = \dfrac{k}{V}$

46. $R = k(T - T_e)$

47. $F = \dfrac{km_1m_2}{r^2}$

48. $R = kS(S - L)$

49. $A = \frac{1}{2}bh$

The area of a triangle is jointly proportional to its base and height.

50. $S = 4\pi r^2$

The surface area of a sphere varies directly as the square of the radius r.

51. $V = \frac{4}{3}\pi r^3$

The volume of a sphere varies directly as the cube of its radius.

52. $V = \pi r^2 h$

The volume of a right circular cylinder is jointly proportional to the height and the square of the radius.

53. $r = \dfrac{d}{t}$

Average speed is directly proportional to the distance and inversely proportional to the time.

54. $\omega = \sqrt{\dfrac{kg}{W}}$

ω varies directly as the square root of g and inversely as the square root of W. (**Note:** The constant of proportionality is $\sqrt{k}$.)

55. $A = kr^2$

$9\pi = k(3)^2$

$\pi = k$

$A = \pi r^2$

56. $y = \dfrac{k}{x}$

$3 = \dfrac{k}{25}$

$75 = k$

$y = \dfrac{75}{x}$

57. $y = \dfrac{k}{x}$

$7 = \dfrac{k}{4}$

$28 = k$

$y = \dfrac{28}{x}$

58. $z = kxy$

$64 = k(4)(8)$

$2 = k$

$z = 2xy$

59. $F = krs^3$

$4158 = k(11)(3)^3$

$k = 14$

$F = 14rs^3$

60. $P = \dfrac{kx}{y^2}$

$\dfrac{28}{3} = \dfrac{k(42)}{9^2}$

$\dfrac{28}{3} \cdot \dfrac{81}{42} = k$

$\dfrac{2 \cdot 27}{3} = k$

$18 = k$

$P = \dfrac{18x}{y^2}$

61. $z = \dfrac{kx^2}{y}$

$6 = \dfrac{k(6)^2}{4}$

$\dfrac{24}{36} = k$

$\dfrac{2}{3} = k$

$z = \dfrac{2/3x^2}{y} = \dfrac{2x^2}{3y}$

62. $v = \dfrac{kpq}{s^2}$

$1.5 = \dfrac{k(4.1)(6.3)}{(1.2)^2}$

$\dfrac{(1.5)(1.44)}{(4.1)(6.3)} = k$

$\dfrac{2.16}{25.83} = k$

$k = \dfrac{24}{287}$

$v = \dfrac{24pq}{287s^2}$

63. $d = kv^2$

$0.02 = k\left(\dfrac{1}{4}\right)^2$

$k = 0.32$

$d = 0.32v^2$

$0.12 = 0.32v^2$

$v^2 = \dfrac{0.12}{0.32} = \dfrac{3}{8}$

$v = \dfrac{\sqrt{3}}{2\sqrt{2}} = \dfrac{\sqrt{6}}{4} \approx 0.61$ mi/hr

64. $d = kv^2$

If the velocity is doubled:

$d = k(2v)^2$

$d = k \cdot 4v^2$

$\dfrac{4kv^2}{kv^2} = 4$

d increases by a factor of 4 when velocity is doubled.

65. $r = \dfrac{kl}{A},\ A = \pi r^2 = \dfrac{\pi d^2}{4}$

$r = \dfrac{4kl}{\pi d^2}$

$66.17 = \dfrac{4(1000)k}{\pi\left(\frac{0.0126}{12}\right)^2}$

$k \approx 5.73 \times 10^{-8}$

$r = \dfrac{4(5.73 \times 10^{-8})l}{\pi\left(\frac{0.0126}{12}\right)^2}$

$33.5 = \dfrac{4(5.73 \times 10^{-8})l}{\pi\left(\frac{0.0126}{12}\right)^2}$

$\dfrac{33.5\pi\left(\frac{0.0126}{12}\right)^2}{4(5.73 \times 10^{-8})} = l$

$l \approx 506$ feet

66. From Exercise 65:

$k \approx 5.73 \times 10^{-8}$.

$r = \dfrac{4(5.73 \times 10^{-8})l}{\pi d^2}$

$d = \sqrt{\dfrac{4(5.73 \times 10^{-8})l}{\pi r}}$

$d = \sqrt{\dfrac{4(5.73 \times 10^{-8})(14)}{\pi(0.05)}}$

$d \approx 0.0045$ feet $= 0.054$ inch

67. $W = kmh$

$2116.8 = k(120)(1.8)$

$k = \dfrac{2116.8}{(120)(1.8)} = 9.8$

$W = 9.8mh$

When $m = 100$ kilograms and $h = 1.5$ meters, we have $W = 9.8(100)(1.5) = 1470$ joules.

68. $P = kA = k(\pi r^2) = k\pi\left(\dfrac{d}{2}\right)^2$

$8.78 = k\pi\left(\dfrac{9}{2}\right)^2$

$\dfrac{4(8.78)}{81\pi} = k$

$k \approx 0.138$

However, we do not obtain \$11.78 when $d = 12$ inches.

$P = 0.138\pi\left(\dfrac{12}{2}\right)^2 \approx \$15.61$

Instead, $k = \dfrac{11.78}{36\pi} \approx 0.104.$

For the 15-inch pizza, we have $k = \dfrac{4(14.18)}{225\pi} \approx 0.080.$

The price is not directly proportional to the surface area. The best buy is the 15-inch pizza.

69. $v = \dfrac{k}{A}$

$v = \dfrac{k}{0.75A} = \dfrac{4}{3}\left(\dfrac{k}{A}\right)$

The velocity is increased by one-third.

70. Load $= \dfrac{kwd^2}{l}$

(a) load $= \dfrac{k(2w)d^2}{2l} = \dfrac{kwd^2}{l}$

The safe load is unchanged.

(b) load $= \dfrac{k(2w)(2d)^2}{l} = \dfrac{8kwd^2}{l}$

The safe load is eight times as great.

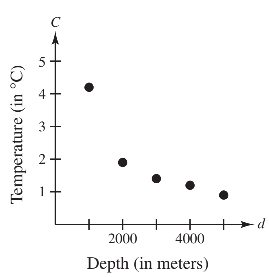
(c) load $= \dfrac{k(2w)(2d)^2}{2l} = \dfrac{4kwd^2}{l}$

The safe load is four times as great.

(d) load $= \dfrac{kw(d/2)^2}{l} = \dfrac{(1/4)kwd^2}{l}$

The safe load is one-fourth as great.

71. (a)



(b) Yes, the data appears to be modeled (approximately) by the inverse proportion model.

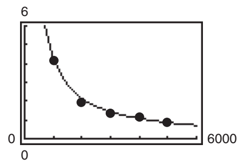
$4.2 = \dfrac{k_1}{1000}$ $\quad 1.9 = \dfrac{k_2}{2000}$ $\quad 1.4 = \dfrac{k_3}{3000}$ $\quad 1.2 = \dfrac{k_4}{4000}$ $\quad 0.9 = \dfrac{k_5}{5000}$

$4200 = k_1$ $\quad 3800 = k_2$ $\quad 4200 = k_3$ $\quad 4800 = k_4$ $\quad 4500 = k_5$

—CONTINUED—

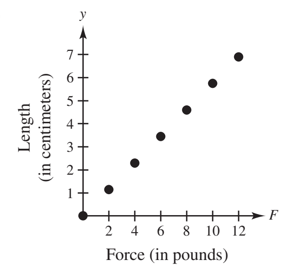
71. —CONTINUED—

(c) Mean: $k = \dfrac{4200 + 3800 + 4200 + 4800 + 4500}{5} = 4300$, Model: $C = \dfrac{4300}{d}$

(d) 

(e) $3 = \dfrac{4300}{d}$

$d = \dfrac{4300}{3} = 1433\frac{1}{3}$ meters

72. (a) 

(b) It appears to fit Hooke's Law.

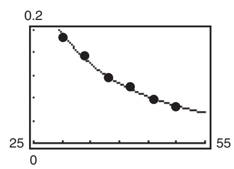
$k \approx \dfrac{6.9}{12} = 0.575$

(c) $y = kF$

$9 = 0.575F$

$F \approx 15.7$ pounds

73. $y = \dfrac{262.76}{x^{2.12}}$

(a) 

(b) $y = \dfrac{262.76}{(25)^{2.12}}$

≈ 0.2857 microwatts per sq. cm.

74. $I = \dfrac{k}{d^2}$

When the distance is doubled:

$I = \dfrac{k}{(2d)^2} = \dfrac{k}{4d^2}$.

The illumination is one-fourth as great. The model given in Exercise 73 is very close to $I = k/d^2$. The difference is probably due to measurement error.

75. False. y will increase if k is positive and y will decrease if k is negative.

76. False. E is jointly proportional (not "directly proportional") to the mass of an object and the square of its velocity.

77. False. The closer the value of $|r|$ is to 1, the better the fit.

78. (a) The data shown could be represented by a linear model which would be a good approximation.

(b) The points do not follow a linear pattern. A linear model would be a poor approximation. A quadratic model would be better.

(c) The points do not follow a linear pattern. A linear model would be a poor approximation.

(d) The data shown could be represented by a linear model which would be a good approximation.

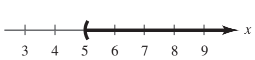
79. The accuracy of the model in predicting prize winnings is questionable because the model is based on limited data.

80. Answers will vary.

81. $3x + 2 > 17$

$3x > 15$

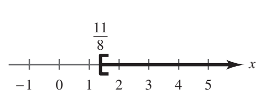
$x > 5$



82. $-7x + 10 \leq -1 + x$

$-8x \leq -11$

$x \geq \frac{11}{8}$

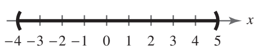


83. $|2x - 1| < 9$

$-9 < 2x - 1 < 9$

$-8 < 2x < 10$

$-4 < x < 5$



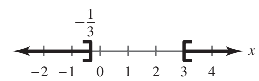
84. $|4 - 3x| + 7 \geq 12$

$|4 - 3x| \geq 5$

$4 - 3x \leq -5$ or $4 - 3x \geq 5$

$-3x \leq -9$ or $-3x \geq 1$

$x \geq 3$ or $x \leq -\frac{1}{3}$



85. $f(x) = \dfrac{x^2 + 5}{x - 3}$

(a) $f(0) = \dfrac{0^2 + 5}{0 - 3} = -\dfrac{5}{3}$

(b) $f(-3) = \dfrac{(-3)^2 + 5}{-3 - 3} = \dfrac{14}{-6} = -\dfrac{7}{3}$

(c) $f(4) = \dfrac{4^2 + 5}{4 - 3} = 21$

86. $f(x) = \begin{cases} -x^2 + 10, & x \geq -2 \\ 6x^2 - 1, & x < -2 \end{cases}$

(a) $f(-2) = -(-2)^2 + 10 = -4 + 10 = 6$

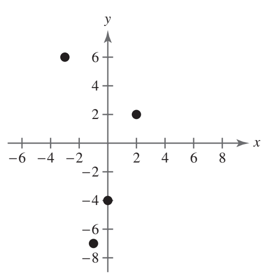
(b) $f(1) = -(1)^2 + 10 = -1 + 10 = 9$

(c) $f(-8) = 6(-8)^2 - 1 = 384 - 1 = 383$

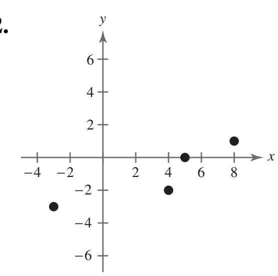
87. Answers will vary.

Review Exercises for Chapter 1

1.



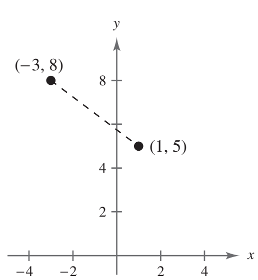
2.



3. $x > 0$ and $y = -2$ in Quadrant IV.

4. $y > 0$ in Quadrants I and II.

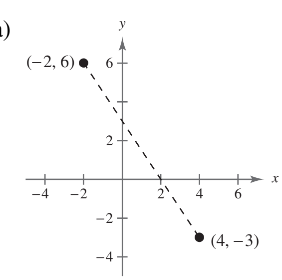
5. (a)



(b) $d = \sqrt{(-3 - 1)^2 + (8 - 5)^2} = \sqrt{16 + 9} = 5$

(c) Midpoint: $\left(\dfrac{-3 + 1}{2}, \dfrac{8 + 5}{2}\right) = \left(-1, \dfrac{13}{2}\right)$

6. (a)

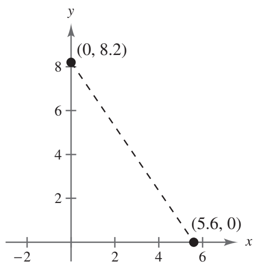


(b) $d = \sqrt{(-2 - 4)^2 + (6 + 3)^2}$

$= \sqrt{36 + 81} = \sqrt{117} = 3\sqrt{13}$

(c) Midpoint: $\left(\dfrac{-2 + 4}{2}, \dfrac{6 - 3}{2}\right) = \left(1, \dfrac{3}{2}\right)$

7. (a)

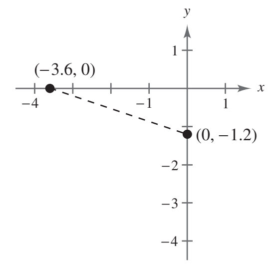


(b) $d = \sqrt{(5.6 - 0)^2 + (0 - 8.2)^2}$

$= \sqrt{31.36 + 67.24} = \sqrt{98.6} \approx 9.9$

(c) Midpoint: $\left(\dfrac{0 + 5.6}{2}, \dfrac{8.2 + 0}{2}\right) = (2.8, 4.1)$

8. (a)



(b) $d = \sqrt{(0 + 3.6)^2 + (-1.2 - 0)^2}$

$= \sqrt{14.4} \approx 3.8$

(c) $\left(\dfrac{0 - 3.6}{2}, \dfrac{-1.2 + 0}{2}\right) = (-1.8, -0.6)$

9. $(4 - 2, 8 - 3) = (2, 5)$

$(6 - 2, 8 - 3) = (4, 5)$

$(4 - 2, 3 - 3) = (2, 0)$

$(6 - 2, 3 - 3) = (4, 0)$

10. Original: $(0, 1), (3, 3), (0, 5), (-3, 3)$

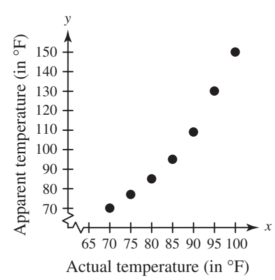
New: $(0 - 4, 1 + 5), (3 - 4, 3 + 5), (0 - 4, 5 + 5), (-3 - 4, 3 + 5) = (-4, 6), (-1, 8), (-4, 10), (-7, 8)$

11. $(2001, 539.1), (2003, 773.8)$

$\left(\dfrac{2001 + 2003}{2}, \dfrac{539.1 + 773.8}{2}\right) = (2002, 656.45)$

In 2002, the sales were approximately $656.45 million.

12. (a)



(b) Change in apparent temperature $= 150°\text{F} - 70°\text{F}$

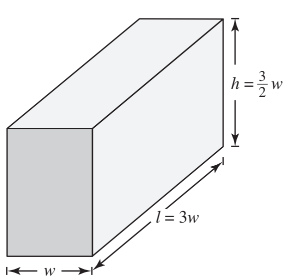
$= 80°\text{F}$

13. $\dfrac{4}{3}\pi r^3 = 47{,}712.94$

$r = \sqrt[3]{\dfrac{47{,}712.94(3)}{4\pi}}$

$r \approx 22.5$ centimeters

14. (a)



(b) $V = l \cdot w \cdot h$

$2304 = (3w) \cdot w \cdot \left(\tfrac{3}{2}w\right)$

$2304 = \tfrac{9}{2}w^3$

$512 = w^3$

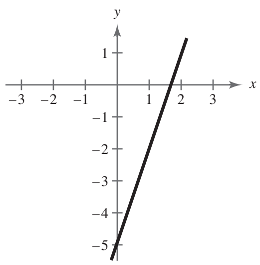
$8 = w \implies w = 8$ inches

$l = 3(8) = 24$ inches

$h = \tfrac{3}{2}(8) = 12$ inches

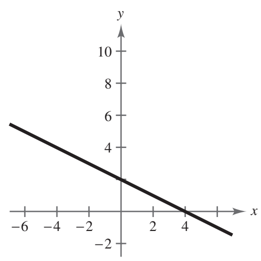
15. $y = 3x - 5$

x	-2	-1	0	1	2
y	-11	-8	-5	-2	1



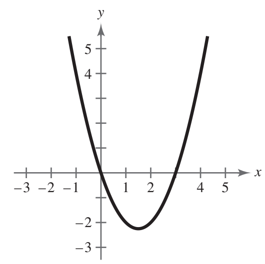
16. $y = -\frac{1}{2}x + 2$

x	-4	-2	0	2	4
y	4	3	2	1	0



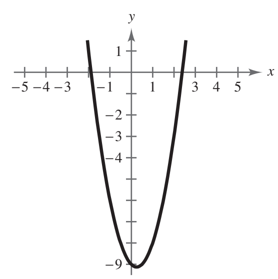
17. $y = x^2 - 3x$

x	-1	0	1	2	3	4
y	4	0	-2	-2	0	4



18. $y = 2x^2 - x - 9$

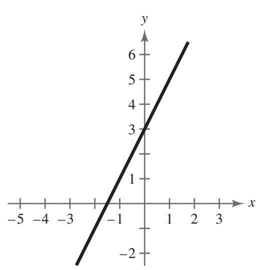
x	-2	-1	0	1	2	3
y	1	-6	-9	-8	-3	6



19. $y - 2x - 3 = 0$

$y = 2x + 3$

Line with x-intercept $\left(-\frac{3}{2}, 0\right)$ and y-intercept $(0, 3)$

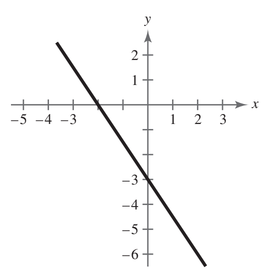


20. $3x + 2y + 6 = 0$

$$2y = -3x - 6$$

$$y = -\frac{3}{2}x - 3$$

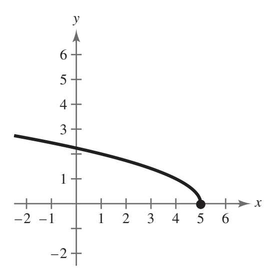
Line with x-intercept $(-2, 0)$ and y-intercept $(0, -3)$



21. $y = \sqrt{5 - x}$

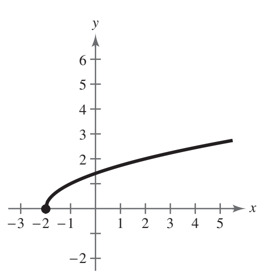
Domain: $(-\infty, 5]$

x	5	4	1	-4
y	0	1	2	3



22. $y = \sqrt{x + 2}$, domain: $[-2, \infty)$

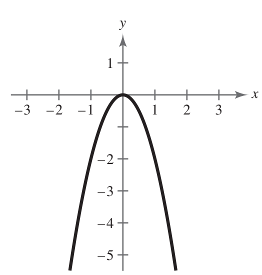
x	-2	0	2	7
y	0	$\sqrt{2}$	2	3



23. $y + 2x^2 = 0$

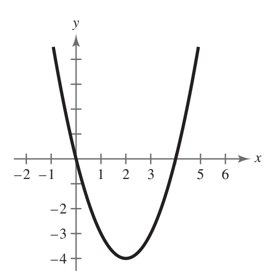
$y = -2x^2$ is a parabola.

x	0	± 1	± 2
y	0	-2	-8



24. $y = x^2 - 4x$ is a parabola.

x	-1	0	1	2	3	4
y	5	0	-3	-4	-3	0



25. $y = 2x + 7$

x-intercept: Let $y = 0$.

$0 = 2x + 7$

$x = -\frac{7}{2}$

$\left(-\frac{7}{2}, 0\right)$

y-intercept: Let $x = 0$.

$y = 2(0) + 7$

$y = 7$

$(0, 7)$

26. $y = |x + 1| - 3$

$0 = |x + 1| - 3$

For $x + 1 > 0$, $0 = x + 1 - 3$, or $2 = x$.

For $x + 1 < 0$, $0 = -(x + 1) - 3$, or $-4 = x$.

$y = |x + 1| - 3$

$y = |0 + 1| - 3$ or $y = -2$

The x-intercepts are $(2, 0)$ and $(-4, 0)$;
the y-intercept is $(0, -2)$.

27. $y = (x - 3)^2 - 4$

x-intercepts: $0 = (x - 3)^2 - 4 \implies (x - 3)^2 = 4$

$\implies x - 3 = \pm 2$

$\implies x = 3 \pm 2$

$\implies x = 5$ or $x = 1$

y-intercept: $y = (0 - 3)^2 - 4 \implies y = 9 - 4 \implies y = 5$

The x-intercepts are $(1, 0)$ and $(5, 0)$.
The y-intercept is $(0, 5)$.

28. $y = x\sqrt{4 - x^2}$

x-intercepts: $0 = x\sqrt{4 - x^2}$

$x = 0 \qquad \sqrt{4 - x^2} = 0$

$4 - x^2 = 0$

$x = \pm 2$

$(0, 0), (-2, 0), (2, 0)$

y-intercept: $y = 0 \cdot \sqrt{4 - 0} = 0$

$(0, 0)$

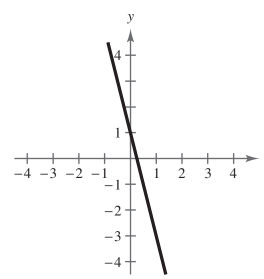
29. $y = -4x + 1$

Intercepts: $\left(\frac{1}{4}, 0\right), (0, 1)$

$y = -4(-x) + 1 \implies y = 4x + 1 \implies$ No y-axis symmetry

$-y = -4x + 1 \implies y = 4x - 1 \implies$ No x-axis symmetry

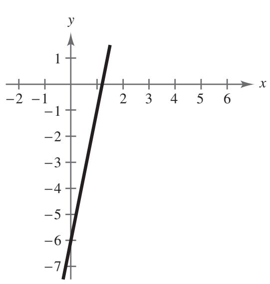
$-y = -4(-x) + 1 \implies y = -4x - 1 \implies$ No origin symmetry



30. $y = 5x - 6$

Intercepts: $\left(\frac{6}{5}, 0\right), (0, -6)$

No symmetry



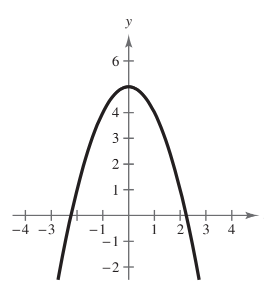
31. $y = 5 - x^2$

Intercepts: $\left(\pm\sqrt{5}, 0\right), (0, 5)$

$y = 5 - (-x)^2 \implies y = 5 - x^2 \implies$ y-axis symmetry

$-y = 5 - x^2 \implies y = -5 + x^2 \implies$ No x-axis symmetry

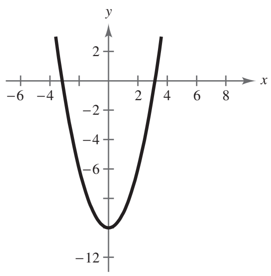
$-y = 5 - (-x)^2 \implies y = -5 + x^2 \implies$ No origin symmetry



32. $y = x^2 - 10$

Intercepts: $(\pm\sqrt{10}, 0), (0, -10)$

y-axis symmetry



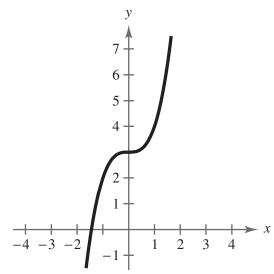
33. $y = x^3 + 3$

Intercepts: $(-\sqrt[3]{3}, 0), (0, 3)$

$y = (-x)^3 + 3 \Rightarrow y = -x^3 + 3 \Rightarrow$ No y-axis symmetry

$-y = x^3 + 3 \Rightarrow y = -x^3 - 3 \Rightarrow$ No x-axis symmetry

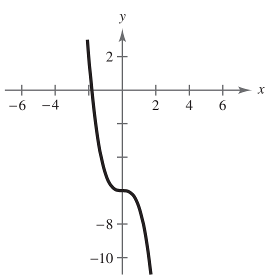
$-y = (-x)^3 + 3 \Rightarrow y = x^3 - 3 \Rightarrow$ No origin symmetry



34. $y = -6 - x^3$

Intercepts: $(\sqrt[3]{-6}, 0), (0, -6)$

No symmetry



35. $y = \sqrt{x + 5}$

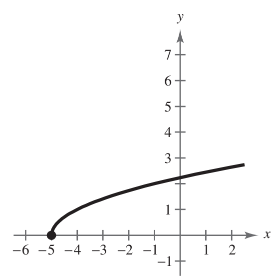
Domain: $[-5, \infty)$

Intercepts: $(-5, 0), (0, \sqrt{5})$

$y = \sqrt{-x + 5} \Rightarrow$ No y-axis symmetry

$-y = \sqrt{x + 5} \Rightarrow y = -\sqrt{x + 5} \Rightarrow$ No x-axis symmetry

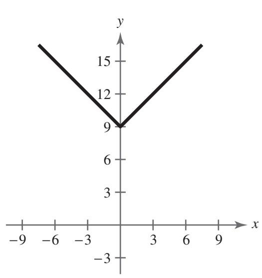
$-y = \sqrt{-x + 5} \Rightarrow y = -\sqrt{-x + 5} \Rightarrow$ No origin symmetry



36. $y = |x| + 9$

Intercepts: $(0, 9)$

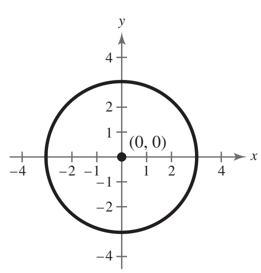
y-axis symmetry



37. $x^2 + y^2 = 9$

Center: $(0, 0)$

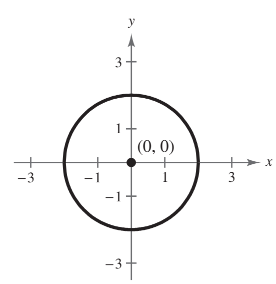
Radius: 3



38. $x^2 + y^2 = 4$

Center: $(0, 0)$

Radius: 2

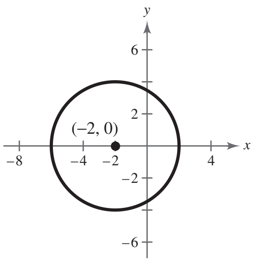


39. $(x+2)^2 + y^2 = 16$

$(x-(-2))^2 + (y-0)^2 = 4^2$

Center: $(-2, 0)$

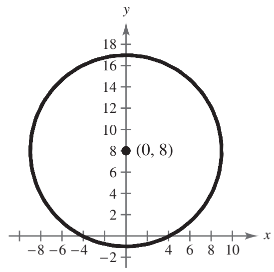
Radius: 4



40. $x^2 + (y-8)^2 = 81$

Center: $(0, 8)$

Radius: 9

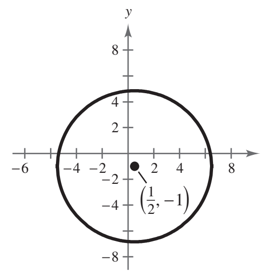


41. $\left(x-\frac{1}{2}\right)^2 + (y+1)^2 = 36$

$\left(x-\frac{1}{2}\right)^2 + (y-(-1))^2 = 6^2$

Center: $\left(\frac{1}{2}, -1\right)$

Radius: 6

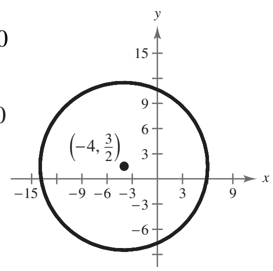


42. $(x+4)^2 + \left(y-\frac{3}{2}\right)^2 = 100$

$(x-(-4))^2 + \left(y-\frac{3}{2}\right)^2 = 100$

Center: $\left(-4, \frac{3}{2}\right)$

Radius: 10



43. Endpoints of a diameter: $(0, 0)$ and $(4, -6)$

Center: $\left(\frac{0+4}{2}, \frac{0+(-6)}{2}\right) = (2, -3)$

Radius: $r = \sqrt{(2-0)^2 + (-3-0)^2} = \sqrt{4+9} = \sqrt{13}$

Standard form: $(x-2)^2 + (y-(-3))^2 = \left(\sqrt{13}\right)^2$

$$(x-2)^2 + (y+3)^2 = 13$$

44. Endpoints of a diameter: $(-2, -3)$ and $(4, -10)$

Center: $\left(\frac{-2+4}{2}, \frac{-3+(-10)}{2}\right) = \left(1, -\frac{13}{2}\right)$

Radius: $r = \sqrt{(1-(-2))^2 + \left(-\frac{13}{2}-(-3)\right)^2} = \sqrt{9+\frac{49}{4}} = \sqrt{\frac{85}{4}}$

Standard form: $(x-1)^2 + \left(y-\left(-\frac{13}{2}\right)\right)^2 = \left(\sqrt{\frac{85}{4}}\right)^2$

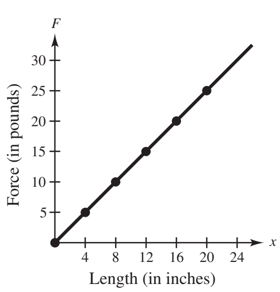
$$(x-1)^2 + \left(y+\frac{13}{2}\right)^2 = \frac{85}{4}$$

45. $F = \frac{5}{4}x,\ 0 \le x \le 20$

(a)

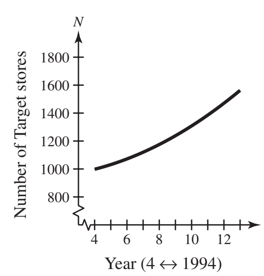
x	0	4	8	12	16	20
F	0	5	10	15	20	25

(b)



(c) When $x = 10$, $F = \frac{50}{4} = 12.5$ pounds.

46. (a)

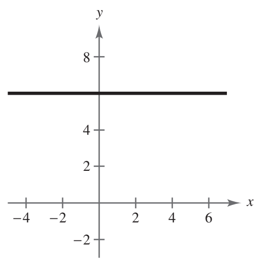


(b) $z = 9.94$; The number of stores was 1300 in 2003.

47. $y = 6$

Horizontal line, $m = 0$

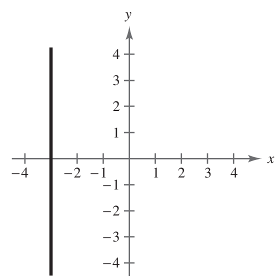
y-intercept: $(0, 6)$



48. $x = -3$

Slope: m is undefined.

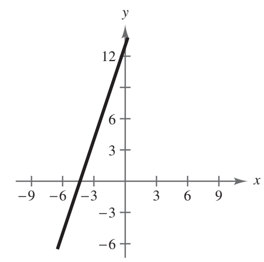
y-intercept: none



49. $y = 3x + 13$

Slope: $m = 3 = \frac{3}{1}$

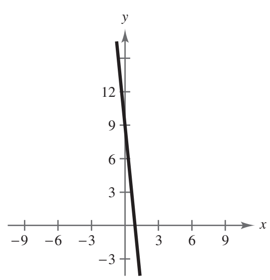
y-intercept: $(0, 13)$



50. $y = -10x + 9$

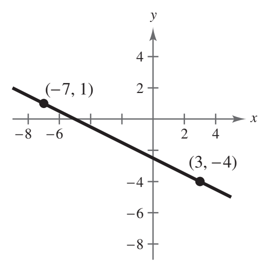
Slope: $m = -10$

y-intercept: $(0, 9)$



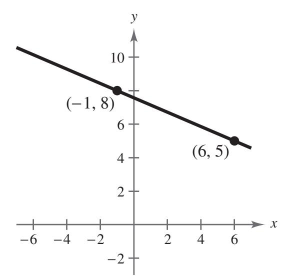
51. $(3, -4), (-7, 1)$

$$m = \frac{1 - (-4)}{-7 - 3} = \frac{5}{-10} = -\frac{1}{2}$$



52. $(-1, 8), (6, 5)$

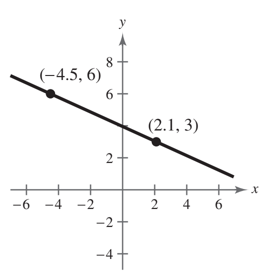
$$m = \frac{5 - 8}{6 - (-1)} = -\frac{3}{7}$$



53. $(-4.5, 6),\ (2.1, 3)$

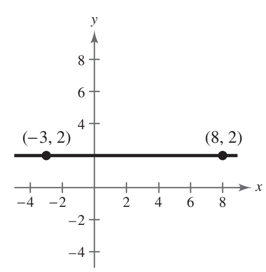
$$m = \frac{3 - 6}{2.1 - (-4.5)}$$

$$= \frac{-3}{6.6} = -\frac{30}{66} = -\frac{5}{11}$$



54. $(-3, 2), (8, 2)$

$$m = \frac{2 - 2}{-3 - 8} = \frac{0}{-11} = 0$$

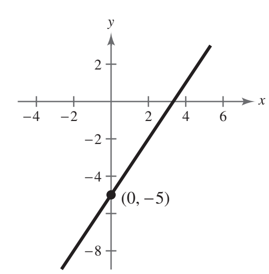


55. $(0, -5), m = \frac{3}{2}$

$$y - (-5) = \frac{3}{2}(x - 0)$$

$$y + 5 = \frac{3}{2}x$$

$$y = \frac{3}{2}x - 5$$

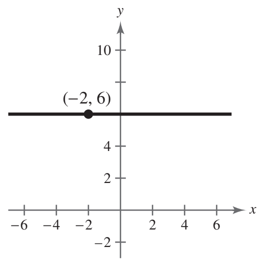


56. $(-2, 6), m = 0$

$y - 6 = 0(x - (-2))$

$y - 6 = 0$

$y = 6$

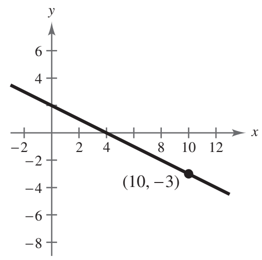


57. $(10, -3), m = -\frac{1}{2}$

$y - (-3) = -\frac{1}{2}(x - 10)$

$y + 3 = -\frac{1}{2}x + 5$

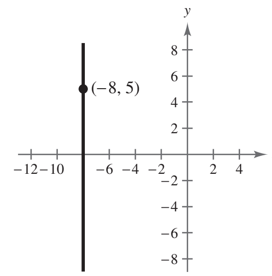
$y = -\frac{1}{2}x + 2$



58. $(-8, 5)$, m is undefined.

The line is vertical.

$x = -8$



59. $(0, 0), (0, 10)$

$$m = \frac{10 - 0}{0 - 0} = \frac{10}{0}, \quad \text{undefined}$$

The line is vertical.

$x = 0$

60. $(2, 5), (-2, -1)$

$$m = \frac{-1 - 5}{-2 - 2} = \frac{-6}{-4} = \frac{3}{2}$$

$$y - 5 = \frac{3}{2}(x - 2)$$

$$2y - 10 = 3x - 6$$

$$2y = 3x + 4$$

$$y = \frac{3}{2}x + 2$$

61. $(-1, 4), (2, 0)$

$$m = \frac{0 - 4}{2 - (-1)} = -\frac{4}{3}$$

$$y - 4 = -\frac{4}{3}(x - (-1))$$

$$y - 4 = -\frac{4}{3}x - \frac{4}{3}$$

$$y = -\frac{4}{3}x + \frac{8}{3}$$

62. $(11, -2), (6, -1)$

$$m = \frac{-1 - (-2)}{6 - 11} = -\frac{1}{5}$$

$$y - (-2) = -\frac{1}{5}(x - 11)$$

$$5y + 10 = -x + 11$$

$$5y = -x + 1$$

$$y = -\frac{1}{5}x + \frac{1}{5}$$

63. Point: $(3, -2)$

$5x - 4y = 8 \implies y = \frac{5}{4}x - 2$ and $m = \frac{5}{4}$

(a) Parallel slope: $m = \frac{5}{4}$

$y - (-2) = \frac{5}{4}(x - 3)$

$y + 2 = \frac{5}{4}x - \frac{15}{4}$

$y = \frac{5}{4}x - \frac{23}{4}$

(b) Perpendicular slope: $m = -\frac{4}{5}$

$y - (-2) = -\frac{4}{5}(x - 3)$

$y + 2 = -\frac{4}{5}x + \frac{12}{5}$

$y = -\frac{4}{5}x + \frac{2}{5}$

64. Point: $(-8, 3)$, $2x + 3y = 5$

$3y = 5 - 2x$

$y = \frac{5}{3} - \frac{2}{3}x$

(a) Parallel slope: $m = -\frac{2}{3}$

$y - 3 = -\frac{2}{3}(x + 8)$

$3y - 9 = -2x - 16$

$3y = -2x - 7$

$y = -\frac{2}{3}x - \frac{7}{3}$

(b) Perpendicular slope: $m = \frac{3}{2}$

$y - 3 = \frac{3}{2}(x + 8)$

$2y - 6 = 3x + 24$

$2y = 3x + 30$

$y = \frac{3}{2}x + 15$

65. $(6, 12{,}500) \quad m = 850$

$$y - 12{,}500 = 850(t - 6)$$
$$y - 12{,}500 = 850t - 5100$$
$$y = 850t + 7400, \quad 6 \le t \le 11$$

66. $(6, 72.95), m = 5.15$

$$V - 72.95 = 5.15(t - 6)$$
$$V - 72.95 = 5.15t - 30.9$$
$$V = 5.15t + 42.05, \quad 6 \le t \le 11$$

67. $16x - y^4 = 0$

$$y^4 = 16x$$
$$y = \pm 2\sqrt[4]{x}$$

No, y is not a function of x. Some x-values correspond to two y-values.

68. $2x - y - 3 = 0$

$$2x - 3 = y$$

Yes, the equation represents y as a function of x.

69. $y = \sqrt{1 - x}$

Yes. Each x-value, $x \le 1$, corresponds to only one y-value so y is a function of x.

70. $|y| = x + 2$ corresponds to $y = x + 2$ or $-y = x + 2$. No, y is not a function of x. Some x-values correspond to two y-values.

71. $f(x) = x^2 + 1$

(a) $f(2) = (2)^2 + 1 = 5$

(b) $f(-4) = (-4)^2 + 1 = 17$

(c) $f(t^2) = (t^2)^2 + 1 = t^4 + 1$

(d) $f(t + 1) = (t + 1)^2 + 1$

$$= t^2 + 2t + 2$$

72. $h(x) = \begin{cases} 2x + 1, & x \le -1 \\ x^2 + 2, & x > -1 \end{cases}$

(a) $h(-2) = 2(-2) + 1 = -3$

(b) $h(-1) = 2(-1) + 1 = -1$

(c) $h(0) = 0^2 + 2 = 2$

(d) $h(2) = 2^2 + 2 = 6$

73. $f(x) = \sqrt{25 - x^2}$

Domain: $25 - x^2 \ge 0$

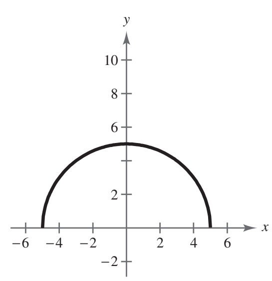
$(5 + x)(5 - x) \ge 0$

Critical numbers: $x = \pm 5$

Test intervals: $(-\infty, -5),\ (-5, 5),\ (5, \infty)$

Test: Is $25 - x^2 \ge 0$?

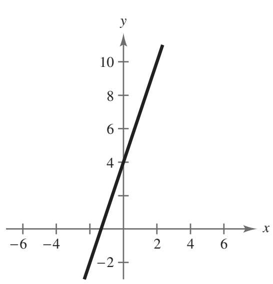
Solution set: $-5 \le x \le 5$



Thus, the domain is all real numbers x such that $-5 \le x \le 5$, or $[-5, 5]$.

74. $f(x) = 3x + 4$

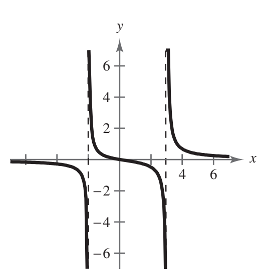
Domain: all real numbers



75. $h(x) = \dfrac{x}{x^2 - x - 6}$

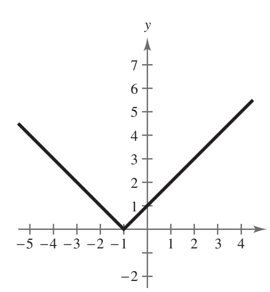
$$= \frac{x}{(x + 2)(x - 3)}$$

Domain: All real numbers x except $x = -2, 3$



76. $h(t) = |t + 1|$

Domain: all real numbers



77. $v(t) = -32t + 48$

(a) $v(1) = 16$ feet per second

(b) $0 = -32t + 48$

$t = \frac{48}{32} = 1.5$ seconds

(c) $v(2) = -16$ feet per second

78. (a) Model: (40% of $(50 - x)$) + (100% of x) = (amount of acid in final mixture)

Amount of acid in final mixture $= f(x)$

$f(x) = 0.4(50 - x) + 1.0x = 20 + 0.6x$

(b) Domain: $0 \le x \le 50$

Range: $20 \le y \le 50$

(c) $20 + 0.6x = 50\%(50)$

$20 + 0.6x = 25$

$0.6x = 5$

$x = 8\frac{1}{3}$ liters

79. $f(x) = 2x^2 + 3x - 1$

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{[2(x+h)^2 + 3(x+h) - 1] - (2x^2 + 3x - 1)}{h}\\ &= \frac{2x^2 + 4xh + 2h^2 + 3x + 3h - 1 - 2x^2 - 3x + 1}{h}\\ &= \frac{h(4x + 2h + 3)}{h}\\ &= 4x + 2h + 3, \quad h \neq 0\end{aligned}$$

80.

$$\begin{aligned}f(x) &= x^3 - 5x^2 + x\\ f(x+h) &= (x+h)^3 - 5(x+h)^2 + (x+h)\\ &= x^3 + 3x^2h + 3xh^2 + h^3 - 5x^2 - 10xh - 5h^2 + x + h\\ \frac{f(x+h) - f(x)}{h} &= \frac{x^3 + 3x^2h + 3xh^2 + h^3 - 5x^2 - 10xh - 5h^2 + x + h - x^3 + 5x^2 - x}{h}\\ &= \frac{3x^2h + 3xh^2 + h^3 - 10xh - 5h^2 + h}{h}\\ &= \frac{h(3x^2 + 3xh + h^2 - 10x - 5h + 1)}{h}\\ &= 3x^2 + 3xh + h^2 - 10x - 5h + 1, \quad h \neq 0\end{aligned}$$

81. $y = (x - 3)^2$

The graph passes the Vertical Line Test. y *is* a function of x.

82. $y = -\frac{3}{5}x^3 - 2x + 1$

A vertical line intersects the graph no more than once, so y *is* a function of x.

83. $x - 4 = y^2$

The graph does not pass the Vertical Line Test. y *is not* a function of x.

84. $x = -|4 - y|$

A vertical line intersects the graph more than once, so y *is not* a function of x.

85. $f(x) = 3x^2 - 16x + 21$

$3x^2 - 16x + 21 = 0$

$(3x - 7)(x - 3) = 0$

$3x - 7 = 0$ or $x - 3 = 0$

$x = \frac{7}{3}$ or $x = 3$

86. $f(x) = 5x^2 + 4x - 1$

$5x^2 + 4x - 1 = 0$

$(5x - 1)(x + 1) = 0$

$5x - 1 = 0 \implies x = \frac{1}{5}$

$x + 1 = 0 \implies x = -1$

87. $f(x) = \frac{8x + 3}{11 - x}$

$\frac{8x + 3}{11 - x} = 0$

$8x + 3 = 0$

$x = -\frac{3}{8}$

88. $f(x) = x^3 - x^2 - 25x + 25$

$x^3 - x^2 - 25x + 25 = 0$

$x^2(x - 1) - 25(x - 1) = 0$

$(x - 1)(x^2 - 25) = 0$

$x - 1 = 0 \implies x = 1$

$x^2 - 25 = 0 \implies x = \pm 5$

89. $f(x) = |x| + |x + 1|$

f is increasing on $(0, \infty)$.

f is decreasing on $(-\infty, -1)$.

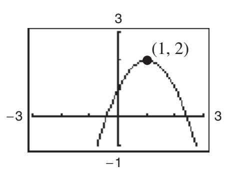
f is constant on $(-1, 0)$.

90. Increasing on $(-2, 0)$ and $(2, \infty)$

Decreasing on $(-\infty, -2)$ and $(0, 2)$

91. $f(x) = -x^2 + 2x + 1$

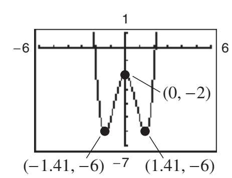
Relative maximum: $(1, 2)$



92. $f(x) = x^4 - 4x^2 - 2$

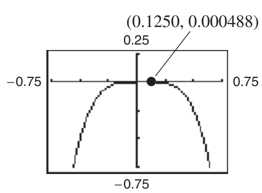
Relative minimum: $(-1.41, -6), (1.41, -6)$

Relative maximum: $(0, -2)$



93. $f(x) = x^3 - 6x^4$

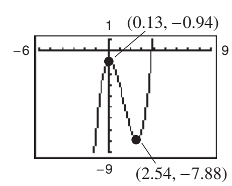
Relative maximum: $(0.125, 0.000488) \approx (0.13, 0.00)$



94. $f(x) = x^3 - 4x^2 + x - 1$

Relative minimum: $(2.54, -7.88)$

Relative maximum: $(0.13, -0.94)$



95. $f(x) = -x^2 + 8x - 4$

$$\frac{f(4) - f(0)}{4 - 0} = \frac{12 - (-4)}{4} = 4$$

The average rate of change of f from $x_1 = 0$ to $x_2 = 4$ is 4.

96. $f(x) = x^3 + 12x - 2,\ x_1 = 0,\ x_2 = 4$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(4) - f(0)}{4 - 0}$$

$$= \frac{110 - (-2)}{4} = \frac{112}{4} = 28$$

The average rate of change from $x = 0$ to $x = 4$ is 28.

97. $f(x) = 2 - \sqrt{x+1}$

$$\frac{f(7) - f(3)}{7 - 3} = \frac{\left(2 - \sqrt{8}\right) - (2 - 2)}{4}$$

$$= \frac{2 - 2\sqrt{2}}{4} = \frac{1 - \sqrt{2}}{2}$$

The average rate of change of f from $x_1 = 3$ to $x_2 = 7$ is $\left(1 - \sqrt{2}\right)/2$.

98. $f(x) = 1 - \sqrt{x+3},\ x_1 = 1,\ x_2 = 6$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(6) - f(1)}{6 - 1}$$

$$= \frac{-2 - (-1)}{5} = \frac{-2 + 1}{5} = -\frac{1}{5} = -0.2$$

The average rate of change from $x = 1$ to $x = 6$ is -0.2.

99. $f(x) = x^5 + 4x - 7$

$$f(-x) = (-x)^5 + 4(-x) - 7$$
$$= -x^5 - 4x - 7$$
$$\neq f(x)$$
$$\neq -f(x)$$

Neither even nor odd

100. $f(x) = x^4 - 20x^2$

$$f(-x) = (-x)^4 - 20(-x)^2 = x^4 - 20x^2 = f(x)$$

The function is even.

101. $f(x) = 2x\sqrt{x^2 + 3}$

$$f(-x) = 2(-x)\sqrt{(-x)^2 + 3}$$
$$= -2x\sqrt{x^2 + 3}$$
$$= -f(x)$$

f is odd.

102. $f(x) = \sqrt[5]{6x^2}$

$$f(-x) = \sqrt[5]{6(-x)^2} = \sqrt[5]{6x^2} = f(x)$$

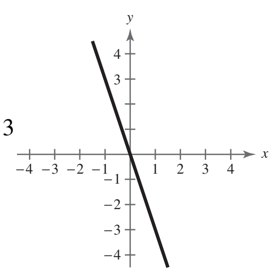
The function is even.

103. $f(2) = -6, f(-1) = 3$

Points: $(2, -6), (-1, 3)$

$$m = \frac{3 - (-6)}{-1 - 2} = \frac{9}{-3} = -3$$

$$y - (-6) = -3(x - 2)$$
$$y + 6 = -3x + 6$$
$$y = -3x$$

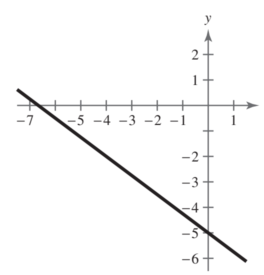


104. $f(0) = -5, f(4) = -8$

$(0, -5), (4, -8)$

$$m = \frac{-8 - (-5)}{4 - 0} = -\frac{3}{4}$$

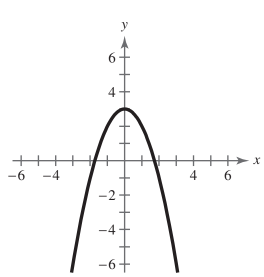
$$y - (-5) = -\frac{3}{4}(x - 0)$$
$$y = -\frac{3}{4}x - 5$$
$$f(x) = -\frac{3}{4}x - 5$$



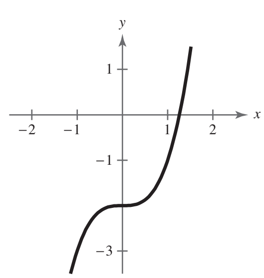
105. $f(x) = 3 - x^2$

Intercepts: $(0, 3), \left(\pm\sqrt{3}, 0\right)$

y-axis symmetry



106. $h(x) = x^3 - 2$

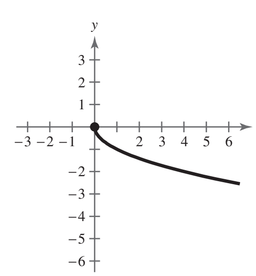


107. $f(x) = -\sqrt{x}$

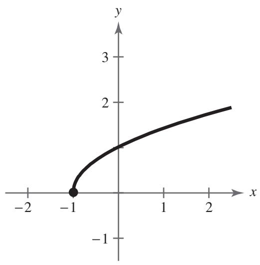
Domain: $x \geq 0$

Intercepts: $(0, 0)$

x	0	1	4	9
y	0	-1	-2	-3



108. $f(x) = \sqrt{x+1}$

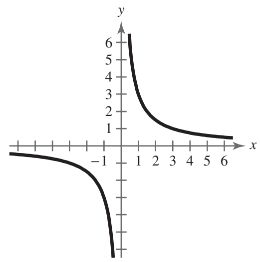


109. $g(x) = \dfrac{3}{x}$

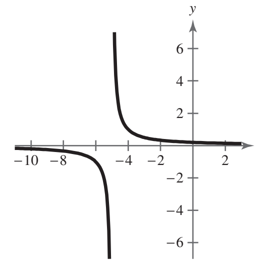
No intercepts

Origin symmetry

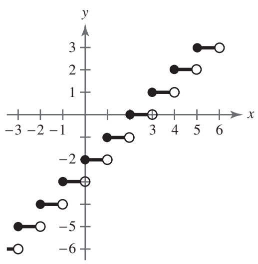
x	-3	-1	1	3
y	-1	-3	3	1



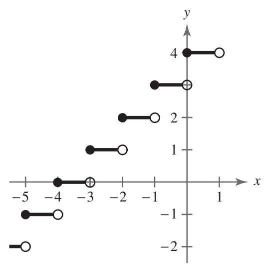
110. $g(x) = \dfrac{1}{x+5}$



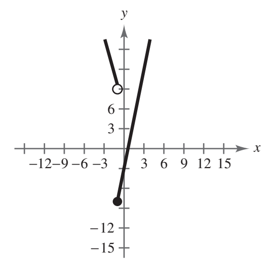
111. $f(x) = [\![x]\!] - 2$



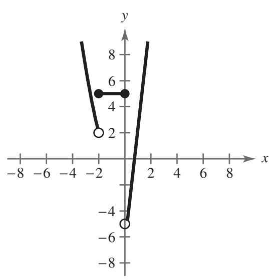
112. $g(x) = [\![x+4]\!]$



113. $f(x) = \begin{cases} 5x - 3, & x \geq -1 \\ -4x + 5, & x < -1 \end{cases}$



114. $f(x) = \begin{cases} x^2 - 2, & x < -2 \\ 5, & -2 \leq x \leq 0 \\ 8x - 5, & x > 0 \end{cases}$



115. Common function: $f(x) = x^3$

Horizontal shift 4 units to the left and a vertical shift 4 units upward

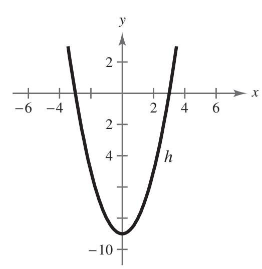
116. The graph of $y = \sqrt{x}$ was shifted upward 4 units.

117. (a) $f(x) = x^2$

(b) $h(x) = x^2 - 9$

Vertical shift 9 units downward

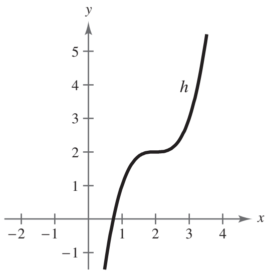
(c)



(d) $h(x) = f(x) - 9$

118. (a) $f(x) = x^3$

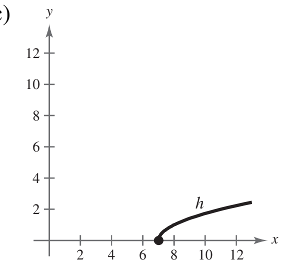
(b) $h(x) = (x - 2)^3 + 2$
Horizontal shift of 2 units to the right; vertical shift of 2 units upward

(c)


(d) $h(x) = f(x - 2) + 2$

119. (a) $f(x) = \sqrt{x}$

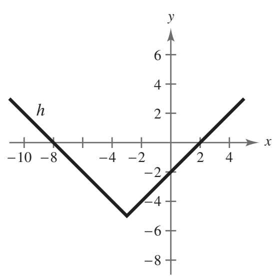
(b) $h(x) = \sqrt{x - 7}$
Horizontal shift 7 units to the right

(c)


(d) $h(x) = f(x - 7)$

120. (a) $f(x) = |x|$

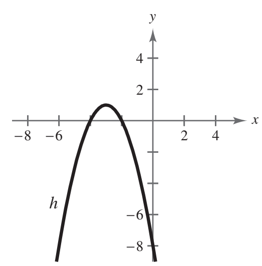
(b) $h(x) = |x + 3| - 5$
Horizontal shift of 3 units to the left;
vertical shift of 5 units downward

(c)


(d) $h(x) = f(x + 3) - 5$

121. (a) $f(x) = x^2$

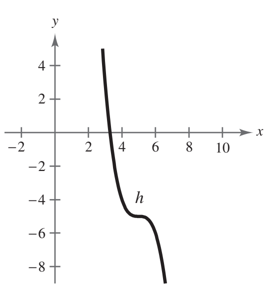
(b) $h(x) = -(x + 3)^2 + 1$
Reflection in the x-axis, a horizontal shift 3 units to the left, and a vertical shift 1 unit upward

(c)


(d) $h(x) = -f(x + 3) + 1$

122. (a) $f(x) = x^3$

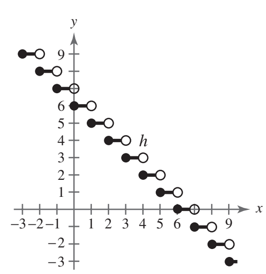
(b) $h(x) = -(x - 5)^3 - 5$
Reflection in the x-axis; horizontal shift of 5 units to the right; vertical shift of 5 units downward

(c)


(d) $h(x) = -f(x - 5) - 5$

123. (a) $f(x) = [\![x]\!]$

(b) $h(x) = -[\![x]\!] + 6$
Reflection in the x-axis and a vertical shift 6 units upward

(c)


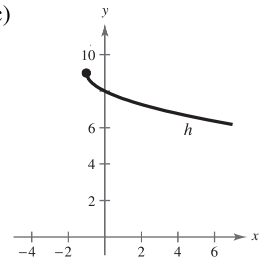
(d) $h(x) = -f(x) + 6$

124. (a) $f(x) = \sqrt{x}$

(b) $h(x) = -\sqrt{x+1} + 9$

Reflection in the x-axis, a horizontal shift 1 unit to the left, and a vertical shift 9 units upward

(c)



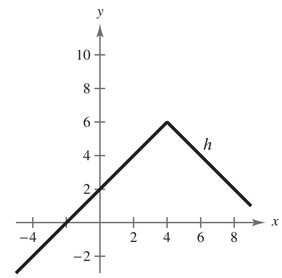
(d) $h(x) = -f(x+1) + 9$

125. (a) $f(x) = |x|$

(b) $h(x) = -|-x+4| + 6$

Reflection in both the x- and y-axes; horizontal shift of 4 units to the right; vertical shift of 6 units upward

(c)



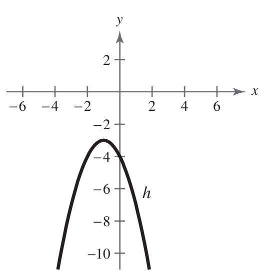
(d) $h(x) = -f(-(x-4)) + 6 = -f(-x+4) + 6$

126. (a) $f(x) = x^2$

(b) $h(x) = -(x+1)^2 - 3$

Reflection in the x-axis; horizontal shift of 1 unit to the left; vertical shift of 3 units downward

(c)



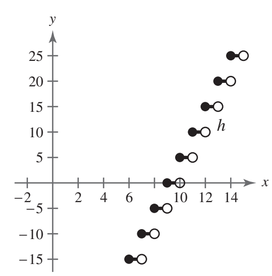
(d) $h(x) = -f(x+1) - 3$

127. (a) $f(x) = [\![x]\!]$

(b) $h(x) = 5[\![x-9]\!]$

Horizontal shift 9 units to the right and a vertical stretch (each y-value is multiplied by 5)

(c)



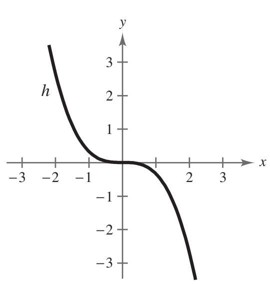
(d) $h(x) = 5f(x-9)$

128. (a) $f(x) = x^3$

(b) $h(x) = -\frac{1}{3}x^3$

Reflection in the x-axis; vertical shrink (each y-value is multiplied by $\frac{1}{3}$)

(c)



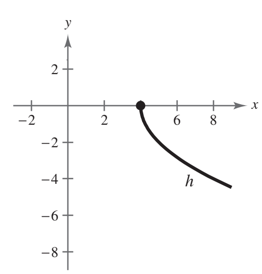
(d) $h(x) = -\frac{1}{3}f(x)$

129. (a) $f(x) = \sqrt{x}$

(b) $h(x) = -2\sqrt{x-4}$

Reflection in the x-axis, a vertical stretch (each y-value is multiplied by 2), and a horizontal shift 4 units to the right

(c)



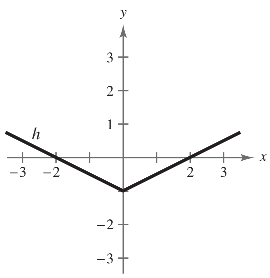
(d) $h(x) = -2f(x-4)$

130. (a) $f(x) = |x|$

(b) $h(x) = \frac{1}{2}|x| - 1$

Vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{2}\right)$; vertical shift of 1 unit downward

(c)



(d) $h(x) = \frac{1}{2}f(x) - 1$

131. $f(x) = x^2 + 3$, $g(x) = 2x - 1$

(a) $(f + g)(x) = (x^2 + 3) + (2x - 1) = x^2 + 2x + 2$

(b) $(f - g)(x) = (x^2 + 3) - (2x - 1) = x^2 - 2x + 4$

(c) $(fg)(x) = (x^2 + 3)(2x - 1) = 2x^3 - x^2 + 6x - 3$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{x^2 + 3}{2x - 1}$, Domain: $x \neq \dfrac{1}{2}$

132. $f(x) = x^2 - 4$, $g(x) = \sqrt{3 - x}$

(a) $(f + g)(x) = f(x) + g(x) = x^2 - 4 + \sqrt{3 - x}$

(b) $(f - g)(x) = f(x) - g(x) = x^2 - 4 - \sqrt{3 - x}$

(c) $(fg)(x) = f(x)g(x) = (x^2 - 4)\left(\sqrt{3 - x}\right)$

(d) $(f/g)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x^2 - 4}{\sqrt{3 - x}}, \quad x < 3$

133. $f(x) = \frac{1}{3}x - 3$, $g(x) = 3x + 1$

The domains of $f(x)$ and $g(x)$ are all real numbers.

(a) $(f \circ g)(x) = f(g(x))$

$$= f(3x + 1)$$
$$= \tfrac{1}{3}(3x + 1) - 3$$
$$= x + \tfrac{1}{3} - 3$$
$$= x - \tfrac{8}{3}$$

Domain: all real numbers

(b) $(g \circ f)(x) = g(f(x))$

$$= g\left(\tfrac{1}{3}x - 3\right)$$
$$= 3\left(\tfrac{1}{3}x - 3\right) + 1$$
$$= x - 9 + 1$$
$$= x - 8$$

Domain: all real numbers

134. $f(x) = x^3 - 4$, $g(x) = \sqrt[3]{x + 7}$

The domains of $f(x)$ and $g(x)$ are all real numbers.

(a) $(f \circ g)(x) = f(g(x))$

$$= \left(\sqrt[3]{x + 7}\right)^3 - 4$$
$$= x + 7 - 4$$
$$= x + 3$$

Domain: all real numbers

(b) $(g \circ f)(x) = g(f(x))$

$$= \sqrt[3]{(x^3 - 4) + 7}$$
$$= \sqrt[3]{x^3 + 3}$$

Domain: all real numbers

135. $h(x) = (6x - 5)^3$

Answer is not unique.

One possibility: Let $f(x) = x^3$ and $g(x) = 6x - 5$.

$f(g(x)) = f(6x - 5) = (6x - 5)^3 = h(x)$

136. $h(x) = \sqrt[3]{x + 2}$

Answer is not unique.

One possibility: Let $g(x) = x + 2$ and $f(x) = \sqrt[3]{x}$.

$f(g(x)) = f(x + 2) = \sqrt[3]{x + 2} = h(x)$

137. $v(t) = -31.86t^2 + 233.6t + 2594$

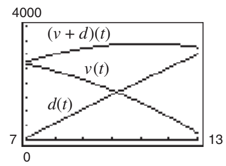
$d(t) = -4.18t^2 + 571.0t - 3706$

(a) $(v + d)(t) = v(t) + d(t)$

$= -36.04t^2 + 804.6t - 1112$

$(v + d)(t)$ represents the combined factory sales (in millions of dollars) for VCRs and DVD players from 1997 to 2003.

(b)



(c) $(v + d)(10) = \$3330$ million

138. (a) $N(T(t)) = 25(2t + 1)^2 - 50(2t + 1) + 300, \quad 2 \le t \le 20$

$= 25(4t^2 + 4t + 1) - 100t - 50 + 300$

$= 100t^2 + 100t + 25 - 100t + 250$

$= 100t^2 + 275$

The composition $N(T(t))$ represents the number of bacteria in the food as a function of time.

(b) When $N = 750$,

$750 = 100t^2 + 275$

$100t^2 = 475$

$t^2 = 4.75$

$t = 2.18$ hours.

After about 2.18 hours, the bacterial count will reach 750.

139. $f(x) = x - 7$

$f^{-1}(x) = x + 7$

$f(f^{-1}(x)) = f(x + 7) = (x + 7) - 7 = x$

$f^{-1}(f(x)) = f^{-1}(x - 7) = (x - 7) + 7 = x$

140. $f(x) = x + 5$

$y = x + 5$

$x = y + 5$

$y = x - 5$

$f^{-1}(x) = x - 5$

$f(f^{-1}(x)) = f(x - 5) = x - 5 + 5 = x$

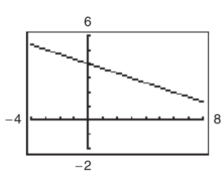
$f^{-1}(f(x)) = f^{-1}(x + 5) = x + 5 - 5 = x$

141. The graph passes the Horizontal Line Test. The function has an inverse.

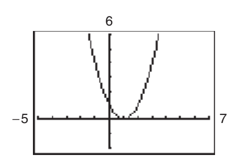
142. No, the function does not have an inverse because some horizontal lines intersect the graph twice.

143. $f(x) = 4 - \frac{1}{3}x$

The graph passes the Horizontal Line Test. The function has an inverse.

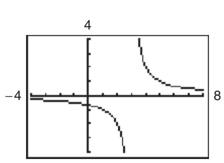


144. No, the function does not have an inverse because some horizontal lines intersect the graph twice.

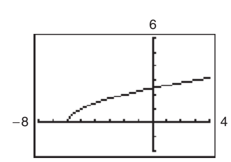


145. $h(t) = \dfrac{2}{t - 3}$

The graph passes the Horizontal Line Test. The function has an inverse.



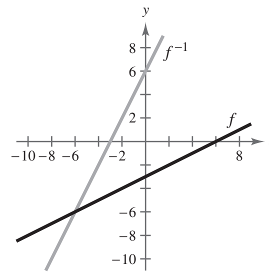
146. Yes, the function has an inverse because no horizontal lines intersect the graph at more than one point.



147. (a)

$$\begin{aligned} f(x) &= \tfrac{1}{2}x - 3 \\ y &= \tfrac{1}{2}x - 3 \\ x &= \tfrac{1}{2}y - 3 \\ x + 3 &= \tfrac{1}{2}y \\ 2(x + 3) &= y \\ f^{-1}(x) &= 2x + 6 \end{aligned}$$

(b)



(c) The graph of f^{-1} is the reflection of the graph of f about the line $y = x$.

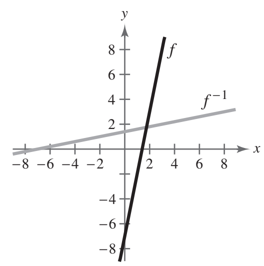
(d) The domains and ranges of f and f^{-1} are the set of all real numbers.

148. $f(x) = 5x - 7$

(a)

$$\begin{aligned} y &= 5x - 7 \\ x &= 5y - 7 \\ x + 7 &= 5y \\ \frac{x + 7}{5} &= y \\ f^{-1}(x) &= \frac{x + 7}{5} \end{aligned}$$

(b)



(c) The graph of f^{-1} is the reflection of the graph of f across the line $y = x$.

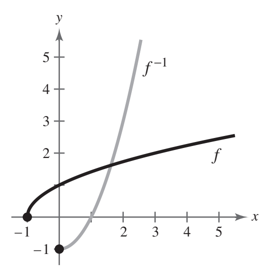
(d) The domains and ranges of f and f^{-1} are the set of all real numbers.

149. (a)

$$\begin{aligned} f(x) &= \sqrt{x + 1} \\ y &= \sqrt{x + 1} \\ x &= \sqrt{y + 1} \\ x^2 &= y + 1 \\ x^2 - 1 &= y \\ f^{-1}(x) &= x^2 - 1,\ x \geq 0 \end{aligned}$$

Note: The inverse must have a restricted domain.

(b)



(c) The graph of f^{-1} is the reflection of the graph of f about the line $y = x$.

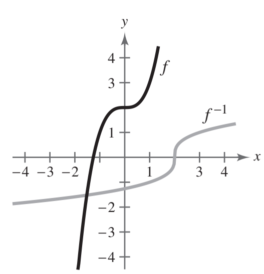
(d) The domain of f and the range of f^{-1} is $[-1, \infty)$. The range of f and the domain of f^{-1} is $[0, \infty)$.

150. $f(x) = x^3 + 2$

(a)

$$\begin{aligned} y &= x^3 + 2 \\ x &= y^3 + 2 \\ x - 2 &= y^3 \\ \sqrt[3]{x - 2} &= y \\ f^{-1}(x) &= \sqrt[3]{x - 2} \end{aligned}$$

(b)



(c) The graph of f^{-1} is the reflection of the graph of f across the line $y = x$.

(d) The domains and ranges of f and f^{-1} are the set of all real numbers.

151. $f(x) = 2(x-4)^2$ is increasing on $[4, \infty)$.

Let $f(x) = 2(x-4)^2,\ x \ge 4$ and $y \ge 0$.

$$y = 2(x-4)^2$$
$$x = 2(y-4)^2,\ x \ge 0,\ y \ge 4$$
$$\frac{x}{2} = (y-4)^2$$
$$\sqrt{\frac{x}{2}} = y - 4$$
$$\sqrt{\frac{x}{2}} + 4 = y$$
$$f^{-1}(x) = \sqrt{\frac{x}{2}} + 4,\ x \ge 0$$

152. $f(x) = |x-2|$

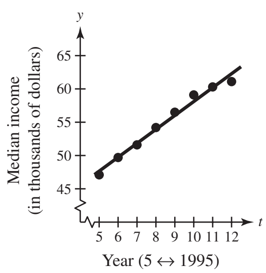
Increasing on $[2, \infty)$

Let $f(x) = x - 2,\ x \ge 2,\ y \ge 0$.

$$y = x - 2$$
$$x = y - 2,\ x \ge 0,\ y \ge 2$$
$$x + 2 = y,\ x \ge 0,\ y \ge 2$$
$$f^{-1}(x) = x + 2,\ x \ge 0$$

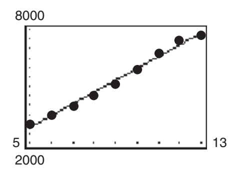
153. $I = 2.09t + 37.2$

(a)



(b) The model is a good fit to the actual data.

154. (a)



(b) $S = 627t - 346$
The model is a good fit to the actual data.

(c) $S = 627.02(18) - 346 = \$10{,}940.36$ million

(d) The factory sales of electronic gaming software in the U.S. increases by $627.02 million each year.

155.
$$D = km$$
$$4 = 2.5k$$
$$1.6 = k$$

In 2 miles:

$D = 1.6(2) = 3.2$ kilometers

In 10 miles:

$D = 1.6(10) = 16$ kilometers

156.
$$P = kS^3$$
$$750 = k(27)^3$$
$$k = 0.03810395$$
$$P = 0.03810395(40)^3$$
$$= 2438.7 \text{ kilowatts}$$

157. $F = ks^2$

If speed is doubled,

$$F = k(2s)^2$$
$$F = 4ks^2.$$

Thus, the force will be changed by a factor of 4.

158.
$$x = \frac{k}{p}$$
$$800 = \frac{k}{5}$$
$$k = 4000$$
$$x = \frac{4000}{6} \approx 667 \text{ boxes}$$

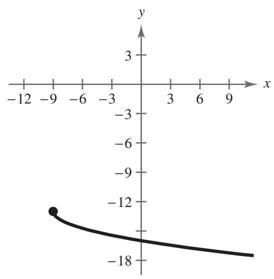
159.
$$T = \frac{k}{r}$$
$$3 = \frac{k}{65}$$
$$k = 3(65) = 195$$
$$T = \frac{195}{r}$$

When $r = 80$ mph,

$$T = \frac{195}{80} = 2.4375 \text{ hours}$$
$$\approx 2 \text{ hours, 26 minutes.}$$

160.
$$C = khw^2$$
$$28.80 = k(16)(6)^2$$
$$k = 0.05$$
$$C = (0.05)(14)(8)^2$$
$$= \$44.80$$

161. False. The graph is reflected in the x-axis, shifted 9 units to the left, then shifted 13 units down.



162. True. If $f(x) = x^3$ and $g(x) = \sqrt[3]{x}$, then the domain of g is all real numbers, which is equal to the range of f and vice versa.

163. True. If $y = kx$, then

$$x = \frac{1}{k}y.$$

164. The Vertical Line Test is used to determine if a graph of y is a function of x. The Horizontal Line Test is used to determine if a function has an inverse function.

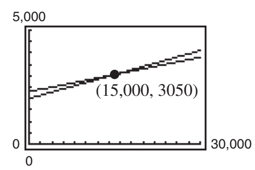
165. A function from a Set A to a Set B is a relation that assigns to each element x in the Set A exactly one element y in the Set B.

Problem Solving for Chapter 1

1. (a) $W_1 = 0.07x + 2000$

(b) $W_2 = 0.05x + 2300$

(c)



Point of intersection: (15,000, 3050)

Both jobs pay the same, \$3050, if you sell \$15,000 per month.

(d) If you think you can sell \$20,000 per month, keep your current job with the higher commission rate. For sales over \$15,000 it pays more than the other job.

2. Mapping numbers onto letters is *not* a function. Each number between 2 and 9 is mapped to more than one letter.

$\{(2, A), (2, B), (2, C), (3, D), (3, E), (3, F), (4, G), (4, H), (4, I), (5, J), (5, K), (5, L),$
$(6, M), (6, N), (6, O), (7, P), (7, Q), (7, R), (7, S), (8, T), (8, U), (8, V), (9, W), (9, X), (9, Y), (9, Z)\}$

Mapping letters onto numbers *is* a function. Each letter is only mapped to one number.

$\{(A, 2), (B, 2), (C, 2), (D, 3), (E, 3), (F, 3), (G, 4), (H, 4), (I, 4), (J, 5), (K, 5), (L, 5),$
$(M, 6), (N, 6), (O, 6), (P, 7), (Q, 7), (R, 7), (S, 7), (T, 8), (U, 8), (V, 8), (W, 9), (X, 9), (Y, 9), (Z, 9)\}$

3. (a) Let $f(x)$ and $g(x)$ be two even functions. Then define $h(x) = f(x) \pm g(x)$.

$$\begin{aligned} h(-x) &= f(-x) \pm g(-x) \\ &= f(x) \pm g(x) \text{ since } f \text{ and } g \text{ are even} \\ &= h(x) \end{aligned}$$

So, $h(x)$ is also even.

(b) Let $f(x)$ and $g(x)$ be two odd functions. Then define $h(x) = f(x) \pm g(x)$.

$$\begin{aligned} h(-x) &= f(-x) \pm g(-x) \\ &= -f(x) \mp g(x) \text{ since } f \text{ and } g \text{ are odd} \\ &= -h(x) \end{aligned}$$

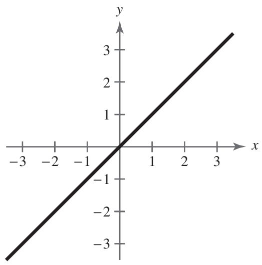
So, $h(x)$ is also odd. (If $f(x) \neq g(x)$)

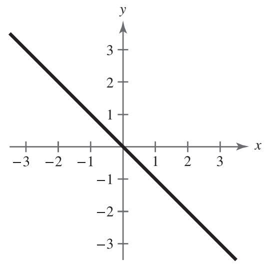
(c) Let $f(x)$ be odd and $g(x)$ be even. Then define $h(x) = f(x) \pm g(x)$.

$$\begin{aligned} h(-x) &= f(-x) \pm g(-x) \\ &= -f(x) \pm g(x) \text{ since } f \text{ is odd and } g \text{ is even} \\ &\neq h(x) \\ &\neq -h(x) \end{aligned}$$

So, $h(x)$ is neither odd nor even.

4. $f(x) = x$ $g(x) = x$





$(f \circ f)(x) = x$ and $(g \circ g)(x) = x$

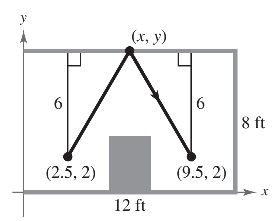
These are the only two linear functions that are their own inverse functions since m has to equal $1/m$ for this to be true.

5.
$$\begin{aligned} f(x) &= a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0 \\ f(-x) &= a_{2n}(-x)^{2n} + a_{2n-2}(-x)^{2n-2} + \cdots + a_2(-x)^2 + a_0 \\ &= a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0 \\ &= f(x) \end{aligned}$$

Therefore, $f(x)$ is even.

6. It appears, from the drawing, that the triangles are equal; thus $(x, y) = (6, 8)$. The line between $(2.5, 2)$ and $(6, 8)$ is $y = \frac{12}{7}x - \frac{16}{7}$. The line between $(9.5, 2)$ and $(6, 8)$ is $y = -\frac{12}{7}x + \frac{128}{7}$. The path of the ball is:

$$f(x) = \begin{cases} \frac{12}{7}x - \frac{16}{7}, & 2.5 \le x \le 6 \\ -\frac{12}{7}x + \frac{128}{7}, & 6 < x \le 9.5 \end{cases}$$



7. (a) April 11: 10 hours

April 12: 24 hours

April 13: 24 hours

April 14: $23\frac{2}{3}$ hours

Total: $81\frac{2}{3}$ hours

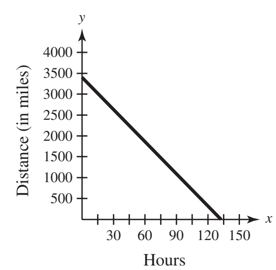
(b) Speed $= \dfrac{\text{distance}}{\text{time}} = \dfrac{2100}{81\frac{2}{3}} = \dfrac{180}{7} = 25\dfrac{5}{7}$ mph

(c) $D = -\dfrac{180}{7}t + 3400$

Domain: $0 \le t \le \dfrac{1190}{9}$

Range: $0 \le D \le 3400$

(d)



8. (a) $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{f(2) - f(1)}{2 - 1} = \dfrac{1 - 0}{1} = 1$

(b) $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{f(1.5) - f(1)}{1.5 - 1} = \dfrac{0.75 - 0}{0.5} = 1.5$

(c) $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{f(1.25) - f(1)}{1.25 - 1} = \dfrac{0.4375 - 0}{0.25} = 1.75$

(d) $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{f(1.125) - f(1)}{1.125 - 1} = \dfrac{0.234375 - 0}{0.125} = 1.875$

(e) $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{f(1.0625) - f(1)}{1.0625 - 1} = \dfrac{0.12109375 - 0}{0.625} = 1.9375$

(f) Yes, the average rate of change appears to be approaching 2.

—CONTINUED—

8. —CONTINUED—

(g) a. $(1, 0), (2, 1), m = 1, y = x - 1$

b. $(1, 0), (1.5, 0.75), m = \dfrac{0.75}{0.5} = 1.5, y = 1.5x - 1.5$

c. $(1, 0), (1.25, 0.4375), m = \dfrac{0.4375}{0.25} = 1.75, y = 1.75x - 1.75$

d. $(1, 0), (1.125, 0.234375), m = \dfrac{0.234375}{0.125} = 1.875, y = 1.875x - 1.875$

e. $(1, 0), (1.0625, 0.12109375), m = \dfrac{0.12109375}{0.0625} = 1.9375, y = 1.9375x - 1.9375$

(h) $(1, f(1)) = (1, 0), m \to 2, y = 2(x - 1), y = 2x - 2$

9. (a)–(d) Use $f(x) = 4x$ and $g(x) = x + 6$.

(a) $(fg)(x) = f(x + 6) = 4(x + 6) = 4x + 24$

(b) $(f \circ g)^{-1}(x) = \dfrac{x - 24}{4} = \dfrac{1}{4}x - 6$

(c) $f^{-1}(x) = \dfrac{1}{4}x$

$g^{-1}(x) = x - 6$

(d) $(g^{-1} \circ f^{-1})(x) = g^{-1}\left(\dfrac{1}{4}x\right) = \dfrac{1}{4}x - 6$

(e) $f(x) = x^3 + 1$ and $g(x) = 2x$

$(f \circ g)(x) = f(2x) = (2x)^3 + 1 = 8x^3 + 1$

$(f \circ g)^{-1}(x) = \sqrt[3]{\dfrac{x - 1}{8}} = \dfrac{1}{2}\sqrt[3]{x - 1}$

$f^{-1}(x) = \sqrt[3]{x - 1}$

$g^{-1}(x) = \dfrac{1}{2}x$

$(g^{-1} \circ f^{-1})(x) = g^{-1}\left(\sqrt[3]{x - 1}\right) = \dfrac{1}{2}\sqrt[3]{x - 1}$

(f) Answers will vary.

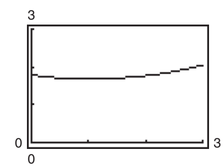
(g) Conjecture: $(f \circ g)^{-1}(x) = (g^{-1} \circ f^{-1})(x)$

10. (a) The length of the trip in the water is $\sqrt{2^2 + x^2}$, and the length of the trip over land is $\sqrt{1 + (3 - x)^2}$. Hence, the total time is

$$T(x) = \frac{\sqrt{4 + x^2}}{2} + \frac{\sqrt{1 + (3 - x)^2}}{4} \text{ hours.}$$

(b) Domain of $T(x)$: $0 \le x \le 3$

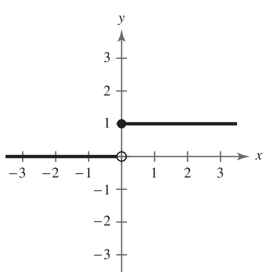
(c)



(d) $T(x)$ is a minimum when $x = 1$.

(e) To reach point Q in the shortest amount of time, you should row to a point one mile down the coast, and then walk the rest of the way.

11. $H(x) = \begin{cases} 1, & x \ge 0 \\ 0, & x < 0 \end{cases}$



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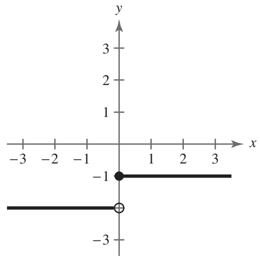
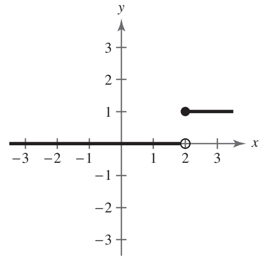
11. —CONTINUED—

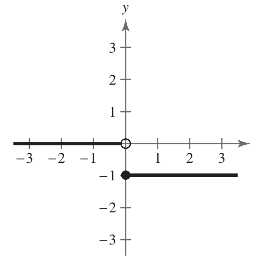
(a) $H(x) - 2$



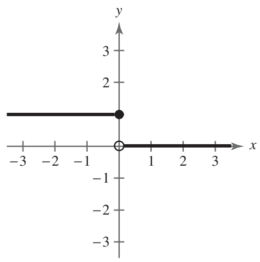
(b) $H(x - 2)$



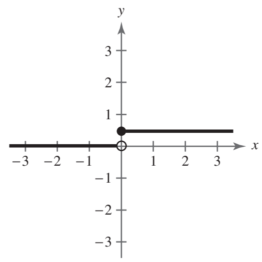
(c) $-H(x)$



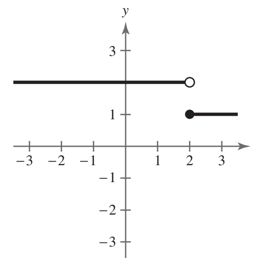
(d) $H(-x)$



(e) $\frac{1}{2}H(x)$



(f) $-H(x - 2) + 2$



12. $f(x) = y = \dfrac{1}{1 - x}$

(a) Domain: all $x \neq 1$

Range: all $y \neq 0$

(b) $f(f(x)) = f\left(\dfrac{1}{1 - x}\right)$

$$= \frac{1}{1 - \left(\dfrac{1}{1 - x}\right)} = \frac{1}{\dfrac{1 - x - 1}{1 - x}}$$

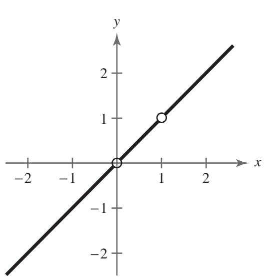
$$= \frac{1 - x}{-x} = \frac{x - 1}{x}$$

Domain: all $x \neq 0, 1$

(c) $f(f(f(x))) = f\left(\dfrac{x - 1}{x}\right) = \dfrac{1}{1 - \left(\dfrac{x - 1}{x}\right)} = \dfrac{1}{\dfrac{1}{x}} = x$

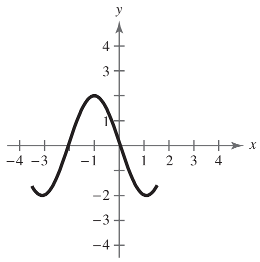
Domain: all $x \neq 0, 1$

The graph is not a line. It has holes at $(0, 0)$ and $(1, 1)$.

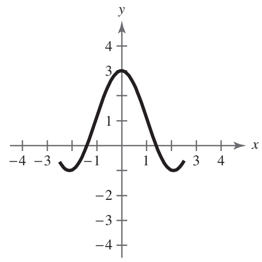


13.
$$\begin{aligned}(f \circ (g \circ h))(x) &= f((g \circ h)(x))\\ &= f(g(h(x)))\\ &= (f \circ g \circ h)(x)\\ ((f \circ g) \circ h)(x) &= (f \circ g)(h(x))\\ &= f(g(h(x)))\\ &= (f \circ g \circ h)(x)\end{aligned}$$

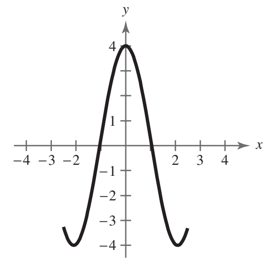
14. (a) $f(x + 1)$



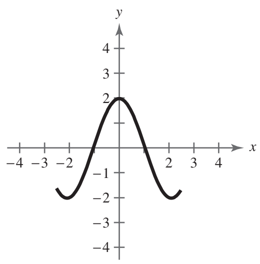
(b) $f(x) + 1$



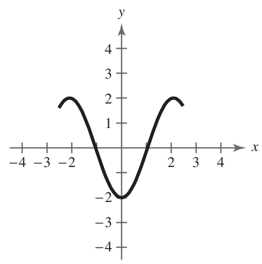
(c) $2f(x)$



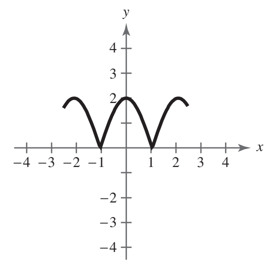
(d) $f(-x)$



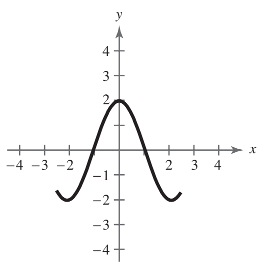
(e) $-f(x)$



(f) $|f(x)|$



(g) $f(|x|)$



15.

x	$f(x)$	$f^{-1}(x)$
-4	—	2
-3	4	1
-2	1	0
-1	0	—
0	-2	-1
1	-3	-2
2	-4	—
3	—	—
4	—	-3

(a)

x	$f(f^{-1}(x))$
-4	$f(f^{-1}(-4)) = f(2) = -4$
-2	$f(f^{-1}(-2)) = f(0) = -2$
0	$f(f^{-1}(0)) = f(-1) = 0$
4	$f(f^{-1}(4)) = f(-3) = 4$

(b)

x	$(f + f^{-1})(x)$
-3	$f(-3) + f^{-1}(-3) = 4 + 1 = 5$
-2	$f(-2) + f^{-1}(-2) = 1 + 0 = 1$
0	$f(0) + f^{-1}(0) = -2 + (-1) = -3$
1	$f(1) + f^{-1}(1) = -3 + (-2) = -5$

(c)

x	$(f \cdot f^{-1})(x)$
-3	$f(-3)f^{-1}(-3) = (4)(1) = 4$
-2	$f(-2)f^{-1}(-2) = (1)(0) = 0$
0	$f(0)f^{-1}(0) = (-2)(-1) = 2$
1	$f(1)f^{-1}(1) = (-3)(-2) = 6$

(d)

x	$\lvert f^{-1}(x)\rvert$
-4	$\lvert f^{-1}(-4)\rvert = \lvert 2\rvert = 2$
-3	$\lvert f^{-1}(-3)\rvert = \lvert 1\rvert = 1$
0	$\lvert f^{-1}(0)\rvert = \lvert -1\rvert = 1$
4	$\lvert f^{-1}(4)\rvert = \lvert -3\rvert = 3$

Chapter 1 Practice Test

1. Given the points $(-3, 4)$ and $(5, -6)$, find (a) the midpoint of the line segment joining the points, and (b) the distance between the points.

2. Graph $y = \sqrt{7 - x}$.

3. Write the standard equation of the circle with center $(-3, 5)$ and radius 6.

4. Find the equation of the line through $(2, 4)$ and $(3, -1)$.

5. Find the equation of the line with slope $m = 4/3$ and y-intercept $b = -3$.

6. Find the equation of the line through $(4, 1)$ perpendicular to the line $2x + 3y = 0$.

7. If it costs a company \$32 to produce 5 units of a product and \$44 to produce 9 units, how much does it cost to produce 20 units? (Assume that the cost function is linear.)

8. Given $f(x) = x^2 - 2x + 1$, find $f(x - 3)$.

9. Given $f(x) = 4x - 11$, find $\dfrac{f(x) - f(3)}{x - 3}$

10. Find the domain and range of $f(x) = \sqrt{36 - x^2}$.

11. Which equations determine y as a function of x?

(a) $6x - 5y + 4 = 0$

(b) $x^2 + y^2 = 9$

(c) $y^3 = x^2 + 6$

12. Sketch the graph of $f(x) = x^2 - 5$.

13. Sketch the graph of $f(x) = |x + 3|$.

14. Sketch the graph of $f(x) = \begin{cases} 2x + 1, & \text{if } x \geq 0, \\ x^2 - x, & \text{if } x < 0. \end{cases}$

15. Use the graph of $f(x) = |x|$ to graph the following:

(a) $f(x + 2)$

(b) $-f(x) + 2$

16. Given $f(x) = 3x + 7$ and $g(x) = 2x^2 - 5$, find the following:

(a) $(g - f)(x)$

(b) $(fg)(x)$

17. Given $f(x) = x^2 - 2x + 16$ and $g(x) = 2x + 3$, find $f(g(x))$.

18. Given $f(x) = x^3 + 7$, find $f^{-1}(x)$.

19. Which of the following functions have inverses?

(a) $f(x) = |x - 6|$

(b) $f(x) = ax + b,\ a \neq 0$

(c) $f(x) = x^3 - 19$

20. Given $f(x) = \sqrt{\dfrac{3 - x}{x}}$, $0 < x \leq 3$, find $f^{-1}(x)$.

Exercises 21–23, true or false?

21. $y = 3x + 7$ and $y = \frac{1}{3}x - 4$ are perpendicular.

22. $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$

23. If a function has an inverse, then it must pass both the Vertical Line Test and the Horizontal Line Test.

24. If z varies directly as the cube of x and inversely as the square root of y, and $z = -1$ when $x = -1$ and $y = 25$, find z in terms of x and y.

25. Use your calculator to find the least square regression line for the data.

x	-2	-1	0	1	2	3
y	1	2.4	3	3.1	4	4.7